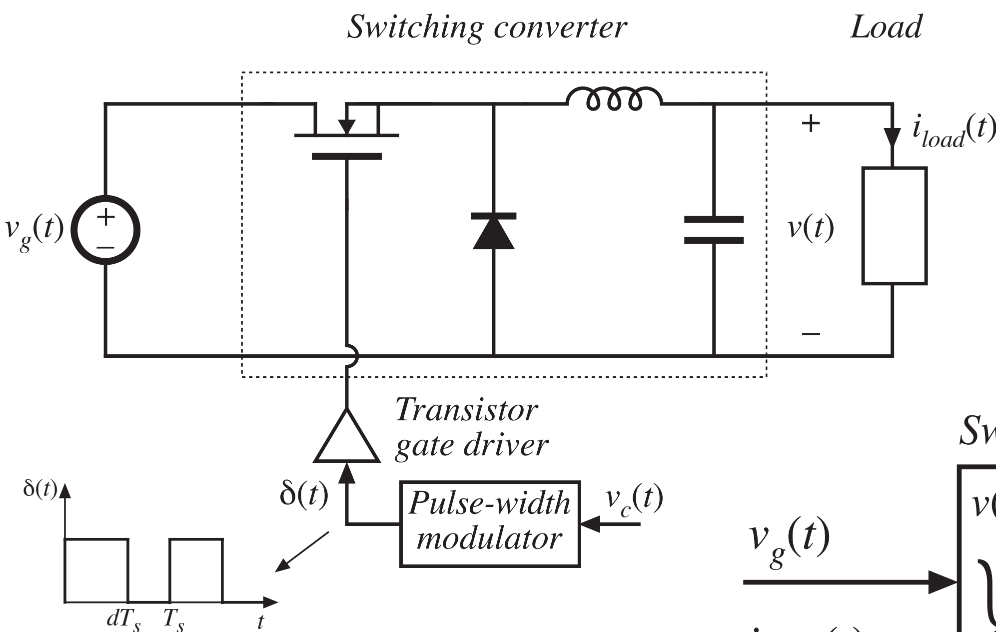
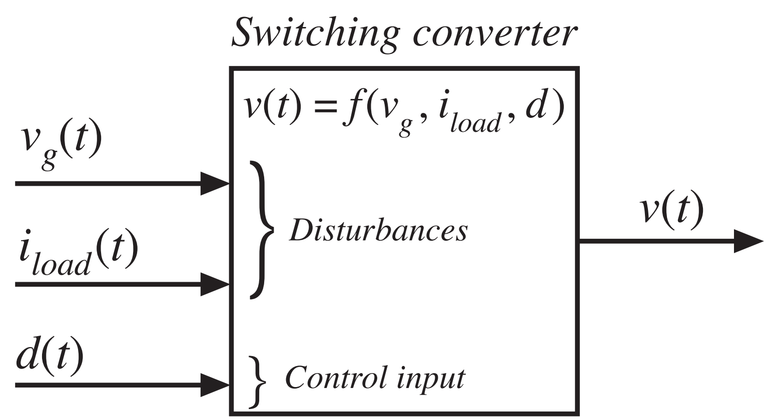
Chapter 9. Controller Design

Controller design

9.1. Introduction



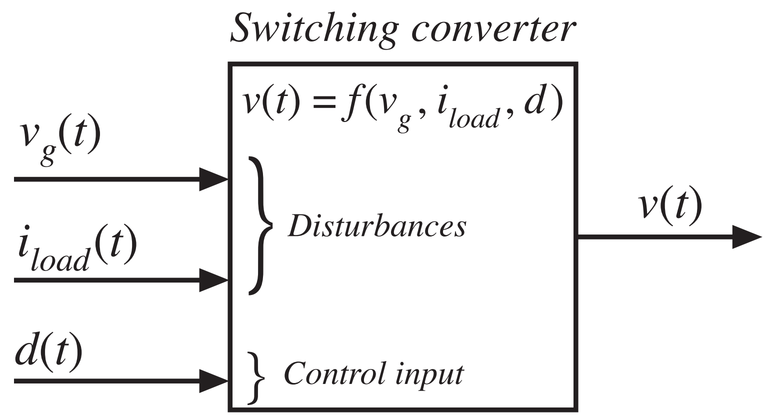
Output voltage of a switching converter depends on duty cycle d, input voltage v_g, and load current i_{load}.



The dc regulator application

Objective: maintain constant output voltage $v(t) = V$, in spite of disturbances in $v_g(t)$ and $i_{load}(t)$.

Typical variation in $v_g(t)$: 100Hz or 120Hz ripple, produced by rectifier circuit.



Load current variations: a significant step-change in load current, such as from 50% to 100% of rated value, may be applied.

A typical output voltage regulation specification: $5\text{V} \pm 0.1\text{V}$.

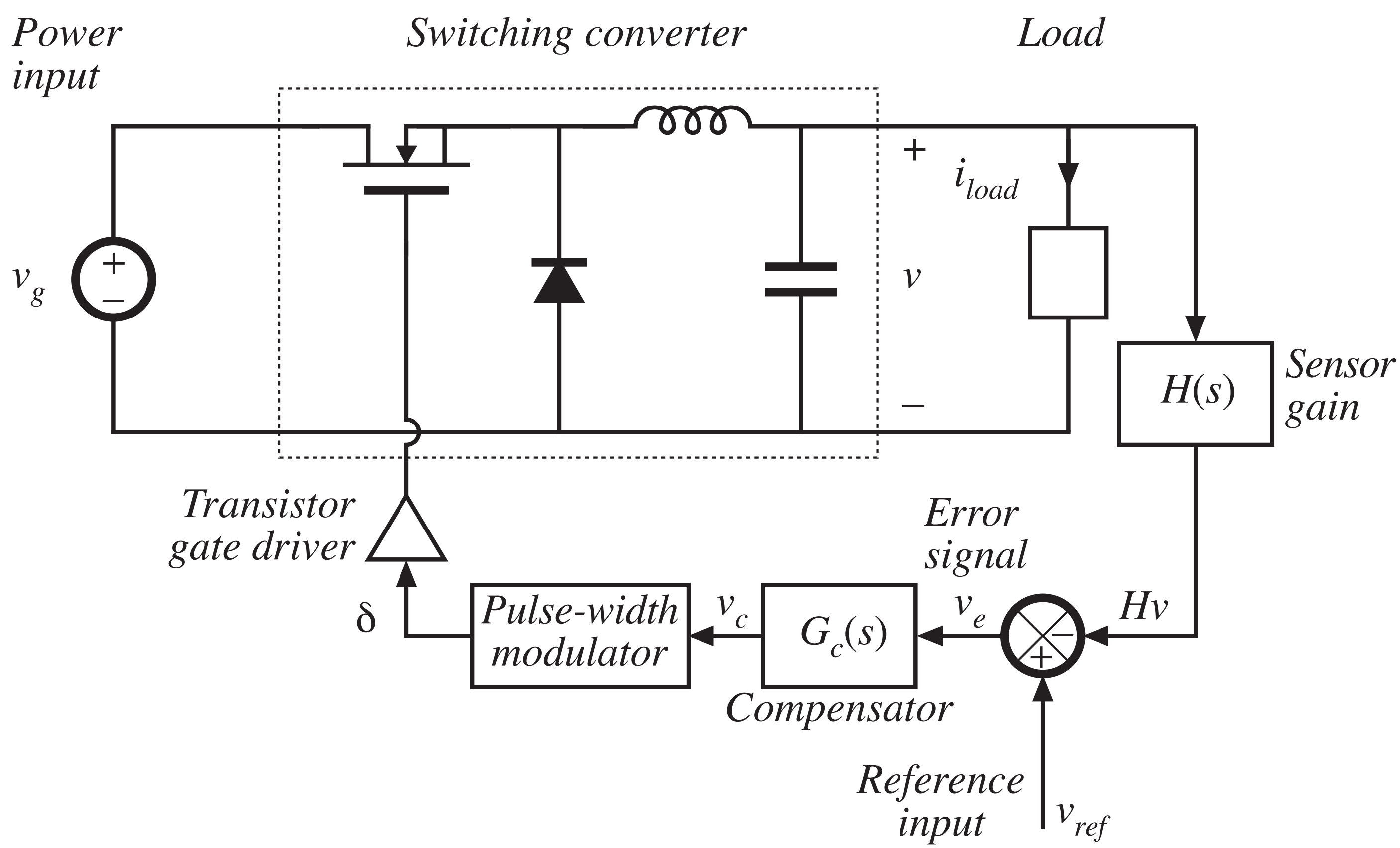
Circuit elements are constructed to some specified tolerance. In high volume manufacturing of converters, all output voltages must meet specifications.

The dc regulator application

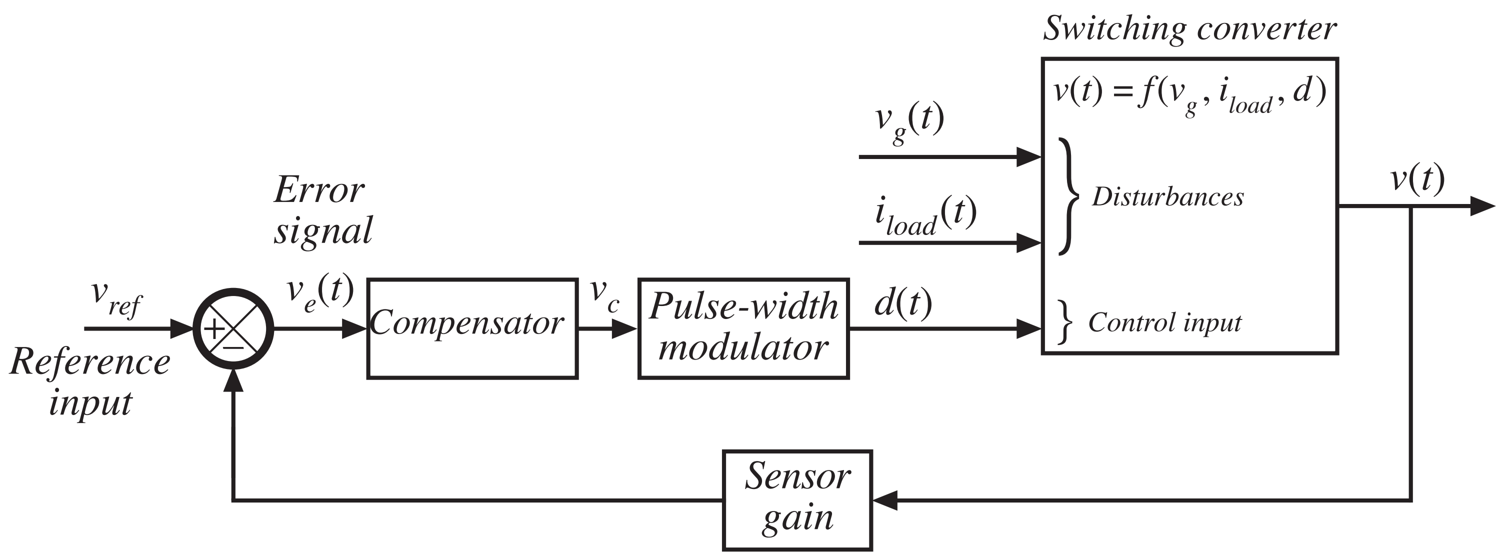
So we cannot expect to set the duty cycle to a single value, and obtain a given constant output voltage under all conditions.

Negative feedback: build a circuit that automatically adjusts the duty cycle as necessary, to obtain the specified output voltage with high accuracy, regardless of disturbances or component tolerances.

Negative feedback:
a switching regulator system

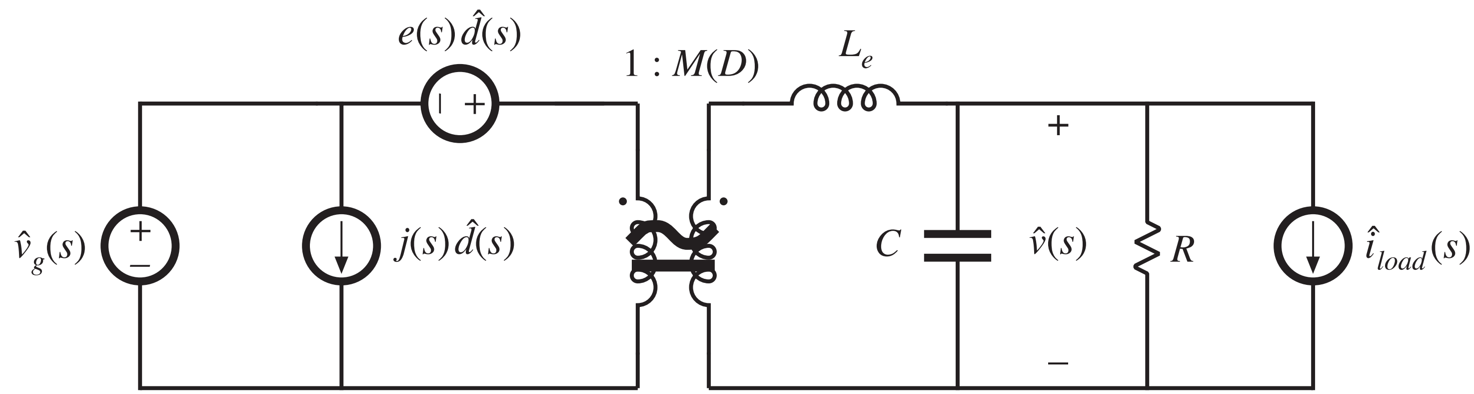


Negative feedback



9.2. Effect of negative feedback on the network transfer functions

Small signal model: open-loop converter



Output voltage can be expressed as

$$\hat{v}(s) = G_{vd}(s)\,\hat{d}(s) + G_{vg}(s)\,\hat{v}_g(s) - Z_{out}(s)\,\hat{i}_{load}(s)$$

where

$$G_{vd}(s) = \left.\frac{\hat{v}(s)}{\hat{d}(s)}\right|_{\substack{\hat{v}_g = 0 \\ \hat{i}_{load} = 0}} \qquad G_{vg}(s) = \left.\frac{\hat{v}(s)}{\hat{v}_g(s)}\right|_{\substack{\hat{d} = 0 \\ \hat{i}_{load} = 0}} \qquad Z_{out}(s) = -\left.\frac{\hat{v}(s)}{\hat{i}_{load}(s)}\right|_{\substack{\hat{d} = 0 \\ \hat{v}_g = 0}}$$

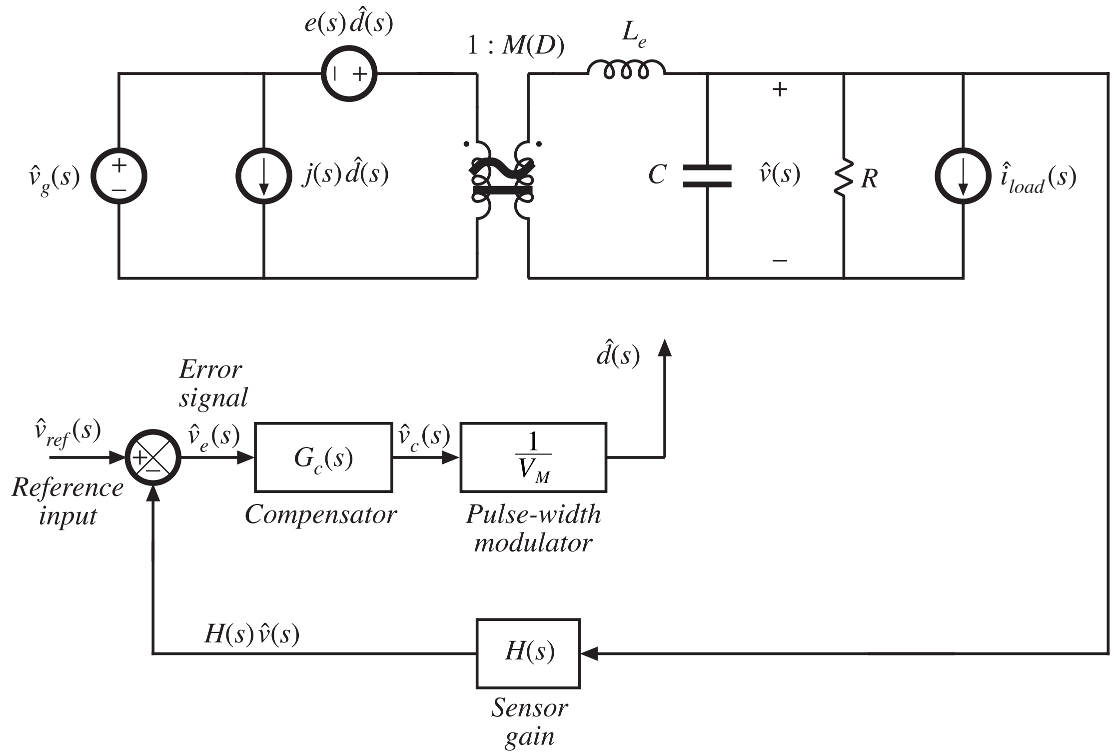
Voltage regulator system small-signal model

- *Use small-signal converter model*
- *Perturb and linearize remainder of feedback loop:*

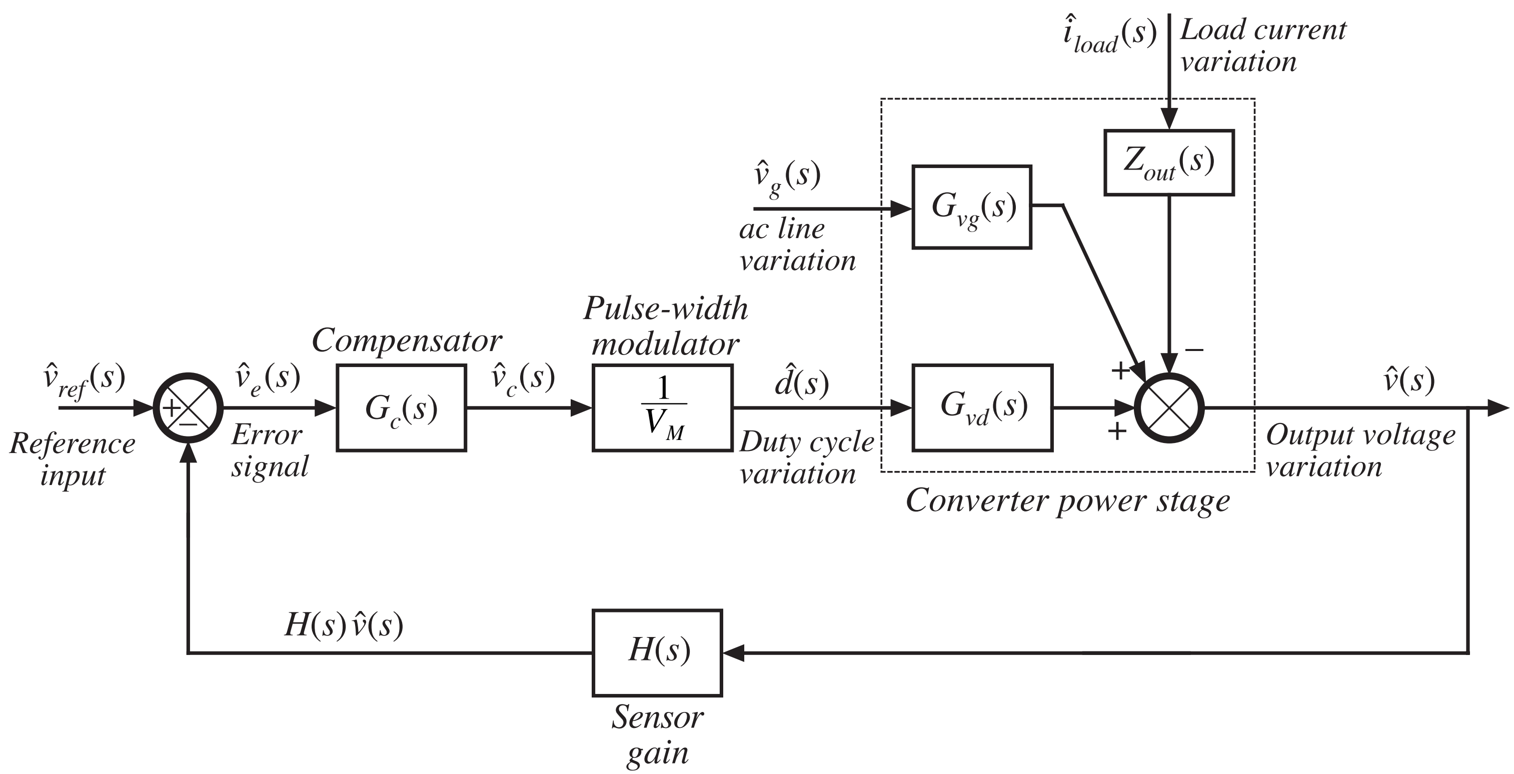
$$v_{ref}(t) = V_{ref} + \hat{v}_{ref}(t)$$

$$v_e(t) = V_e + \hat{v}_e(t)$$

etc.



Regulator system small-signal block diagram



Solution of block diagram

Manipulate block diagram to solve for $\hat{v}(s)$. Result is

$$\hat{v} = \hat{v}_{ref} \frac{G_c G_{vd} / V_M}{1 + H G_c G_{vd} / V_M} + \hat{v}_g \frac{G_{vg}}{1 + H G_c G_{vd} / V_M} - \hat{i}_{load} \frac{Z_{out}}{1 + H G_c G_{vd} / V_M}$$

which is of the form

$$\hat{v} = \hat{v}_{ref} \frac{1}{H} \frac{T}{1 + T} + \hat{v}_g \frac{G_{vg}}{1 + T} - \hat{i}_{load} \frac{Z_{out}}{1 + T}$$

$$\textit{with} \quad T(s) = H(s)\, G_c(s)\, G_{vd}(s) / V_M \quad = \text{"}\textit{loop gain}\text{"}$$

Loop gain $T(s)$ = products of the gains around the negative feedback loop.

9.2.1. Feedback reduces the transfer functions from disturbances to the output

Original (open-loop) line-to-output transfer function:

$$G_{vg}(s) = \left. \frac{\hat{v}(s)}{\hat{v}_g(s)} \right|_{\substack{\hat{d} = 0 \\ \hat{i}_{load} = 0}}$$

With addition of negative feedback, the line-to-output transfer function becomes:

$$\left. \frac{\hat{v}(s)}{\hat{v}_g(s)} \right|_{\substack{\hat{v}_{ref} = 0 \\ \hat{i}_{load} = 0}} = \frac{G_{vg}(s)}{1 + T(s)}$$

Feedback reduces the line-to-output transfer function by a factor of

$$\frac{1}{1 + T(s)}$$

If $T(s)$ is large in magnitude, then the line-to-output transfer function becomes small.

Closed-loop output impedance

Original (open-loop) output impedance:

$$Z_{out}(s) = - \left. \frac{\hat{v}(s)}{\hat{i}_{load}(s)} \right|_{\substack{\hat{d} = 0 \\ \hat{v}_g = 0}}$$

With addition of negative feedback, the output impedance becomes:

$$\left. \frac{\hat{v}(s)}{-\hat{i}_{load}(s)} \right|_{\substack{\hat{v}_{ref} = 0 \\ \hat{v}_g = 0}} = \frac{Z_{out}(s)}{1 + T(s)}$$

Feedback reduces the output impedance by a factor of

$$\frac{1}{1 + T(s)}$$

If $T(s)$ is large in magnitude, then the output impedance is greatly reduced in magnitude.

9.2.2. Feedback causes the transfer function from the reference input to the output to be insensitive to variations in the gains in the forward path of the loop

Closed-loop transfer function from $\hat{v}_{ref}$ to $\hat{v}(s)$ is:

$$\left.\frac{\hat{v}(s)}{\hat{v}_{ref}(s)}\right|_{\substack{\hat{v}_g = 0 \\ \hat{i}_{load} = 0}} = \frac{1}{H(s)} \frac{T(s)}{1 + T(s)}$$

If the loop gain is large in magnitude, i.e., $\| T \| >> 1$, then $(1+T) \approx T$ and $T/(1+T) \approx T/T = 1$. The transfer function then becomes

$$\frac{\hat{v}(s)}{\hat{v}_{ref}(s)} \approx \frac{1}{H(s)}$$

which is independent of the gains in the forward path of the loop.

This result applies equally well to dc values:

$$\frac{V}{V_{ref}} = \frac{1}{H(0)} \frac{T(0)}{1 + T(0)} \approx \frac{1}{H(0)}$$