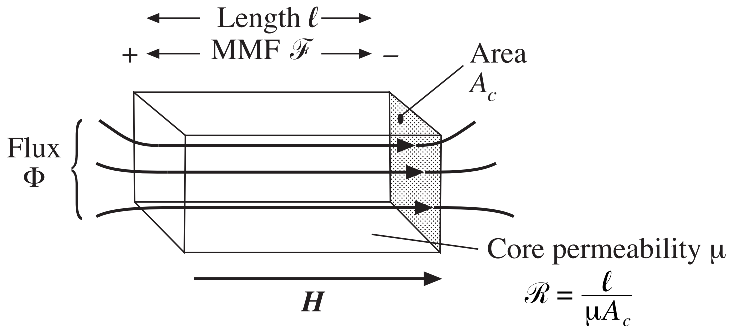
10.1.2 Magnetic circuits

Uniform flux and magnetic field inside a rectangular element:



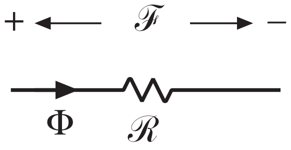
MMF between ends of element is

$$\mathcal{F} = H\ell$$

Since $H = B / \mu$ and $B = \Phi / A_c$, we can express $\mathcal{F}$ as

$$\mathcal{F} = \Phi\mathcal{R} \qquad \text{with} \qquad \mathcal{R} = \frac{\ell}{\mu A_c}$$

A corresponding model:



$\mathcal{R}$ = reluctance of element

Magnetic circuits: magnetic structures composed of multiple windings and heterogeneous elements

- Represent each element with reluctance
- Windings are sources of MMF
- MMF → voltage, flux → current
- Solve magnetic circuit using Kirchoff's laws, etc.

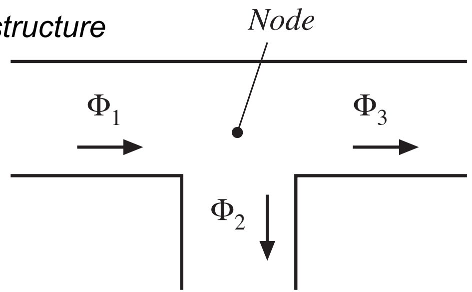
Magnetic analog of Kirchoff's current law

Divergence of $\boldsymbol{B} = 0$

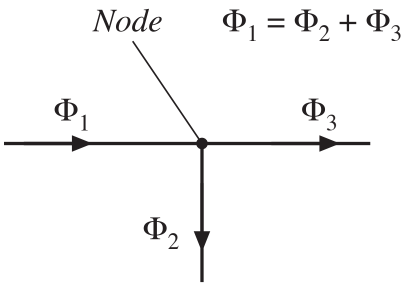
Flux lines are continuous and cannot end

Total flux entering a node must be zero

Physical structure



Magnetic circuit



Magnetic analog of Kirchoff's voltage law

Follows from Ampere's law:

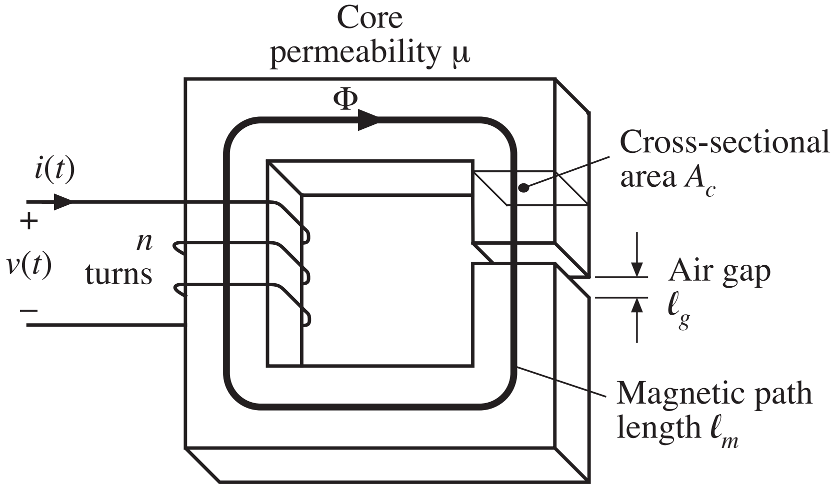
$$\oint_{closed\ path} \boldsymbol{H} \cdot d\boldsymbol{\ell} = \text{total current passing through interior of path}$$

Left-hand side: sum of MMF's across the reluctances around the closed path

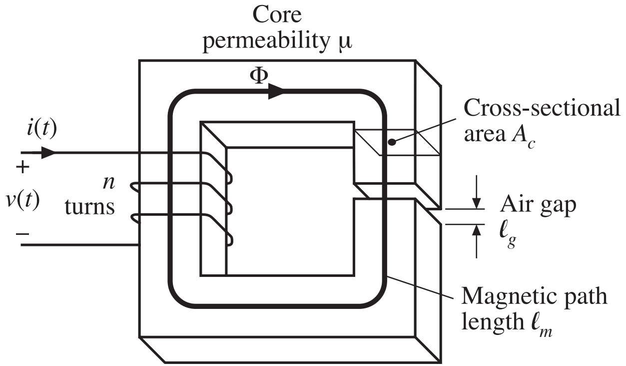
Right-hand side: currents in windings are sources of MMF's. An n-turn winding carrying current $i(t)$ is modeled as an MMF (voltage) source, of value $ni(t)$.

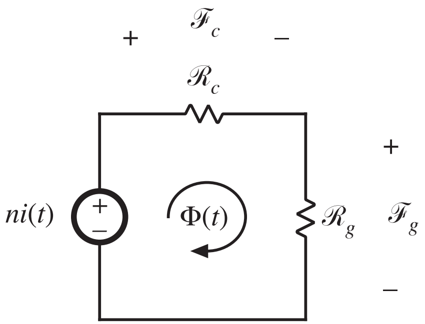
Total MMF's around the closed path add up to zero.

Example: inductor with air gap



Magnetic circuit model





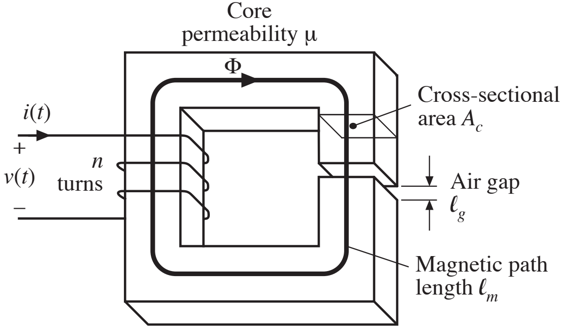
$$\mathcal{F}_c + \mathcal{F}_g = ni$$

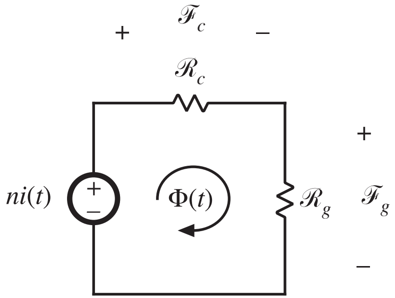
$$\mathcal{R}_c = \frac{\ell_c}{\mu A_c}$$

$$ni = \Phi\left(\mathcal{R}_c + \mathcal{R}_g\right)$$

$$\mathcal{R}_g = \frac{\ell_g}{\mu_0 A_c}$$

Solution of model





$$ni = \Phi\left(\mathcal{R}_c + \mathcal{R}_g\right)$$

$$\mathcal{R}_c = \frac{\ell_c}{\mu A_c}$$

$$\mathcal{R}_g = \frac{\ell_g}{\mu_0 A_c}$$

Faraday's law: $$v(t) = n\frac{d\Phi(t)}{dt}$$

Substitute for Φ: $$v(t) = \frac{n^2}{\mathcal{R}_c + \mathcal{R}_g}\frac{di(t)}{dt}$$

Hence inductance is

$$L = \frac{n^2}{\mathcal{R}_c + \mathcal{R}_g}$$

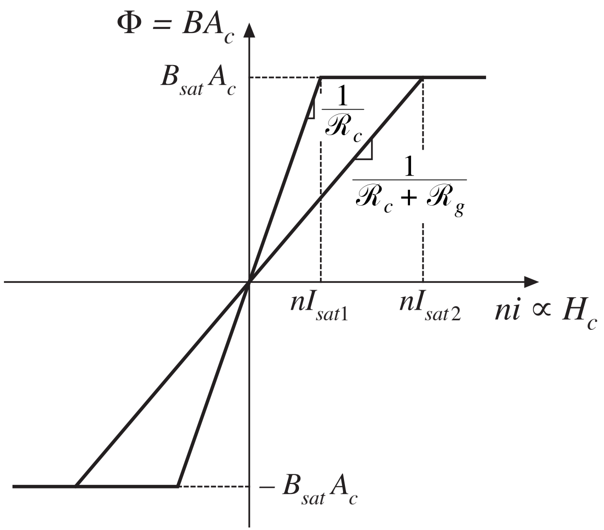
Effect of air gap

$$ni = \Phi\left(\mathcal{R}_c + \mathcal{R}_g\right)$$

$$L = \frac{n^2}{\mathcal{R}_c + \mathcal{R}_g}$$

$$\Phi_{sat} = B_{sat} A_c$$

$$I_{sat} = \frac{B_{sat} A_c}{n}\left(\mathcal{R}_c + \mathcal{R}_g\right)$$



Effect of air gap:

- decrease inductance
- increase saturation current
- inductance is less dependent on core permeability