5.2. Analysis of the conversion ratio *M(D,K)*

Analysis techniques for the discontinuous conduction mode:

Inductor volt-second balance

$$\left\langle v_L \right\rangle = \frac{1}{T_s} \int_0^{T_s} v_L(t)\, dt = 0$$

Capacitor charge balance

$$\left\langle i_C \right\rangle = \frac{1}{T_s} \int_0^{T_s} i_C(t)\, dt = 0$$

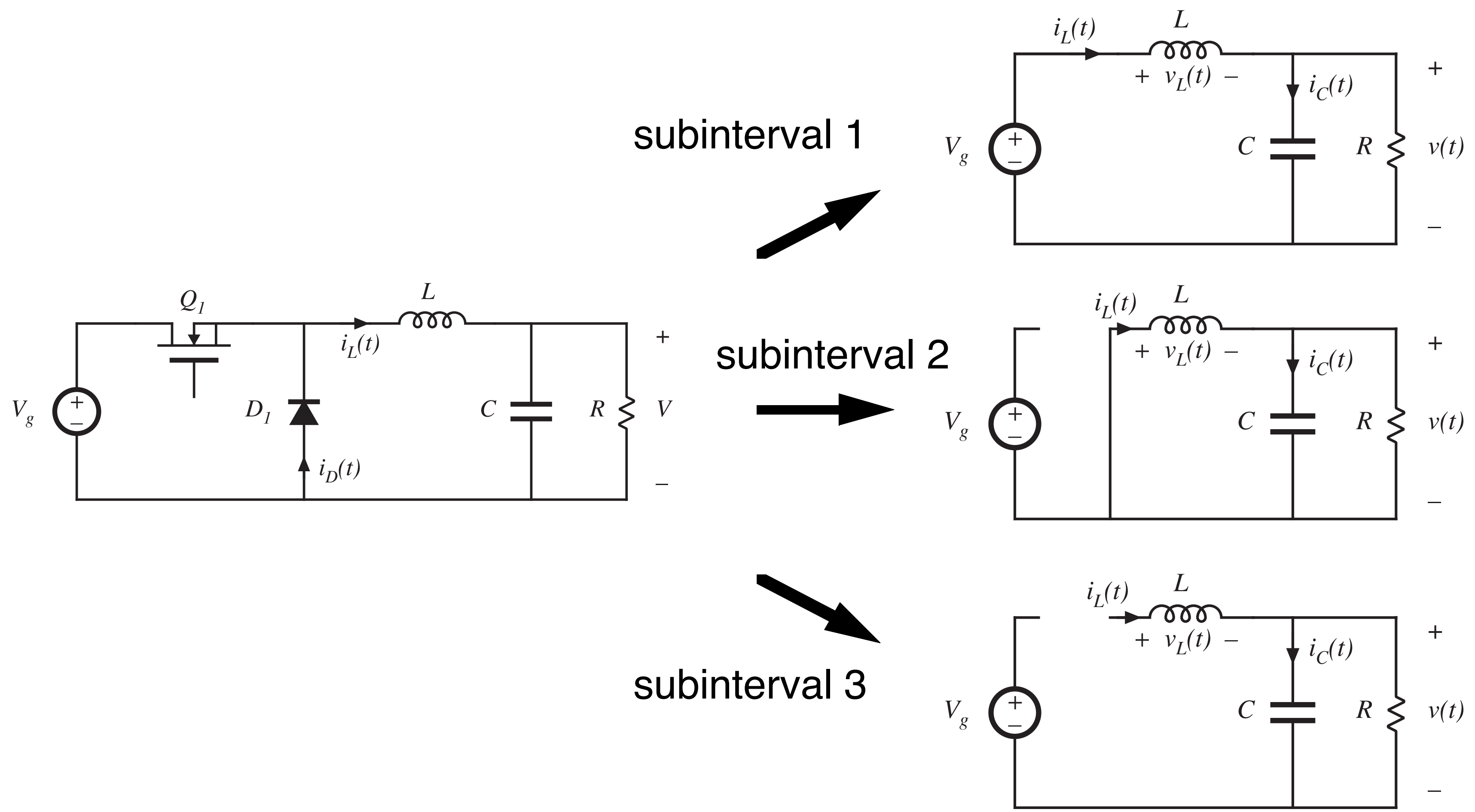
Small ripple approximation sometimes applies:

$$v(t) \approx V \quad \text{because} \quad \Delta v \ll V$$

$$i(t) \approx I \ \text{ is a poor approximation when } \ \Delta i > I$$

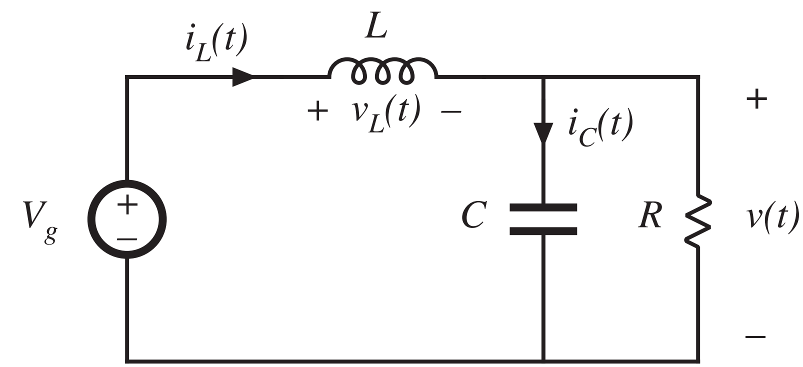
Converter steady-state equations obtained via charge balance on each capacitor and volt-second balance on each inductor. Use care in applying small ripple approximation.

Example: Analysis of DCM buck converter $M(D,K)$



Subinterval 1

$$v_L(t) = V_g - v(t)$$
$$i_C(t) = i_L(t) - v(t) / R$$



Small ripple approximation for $v(t)$ (but not for $i(t)$!):

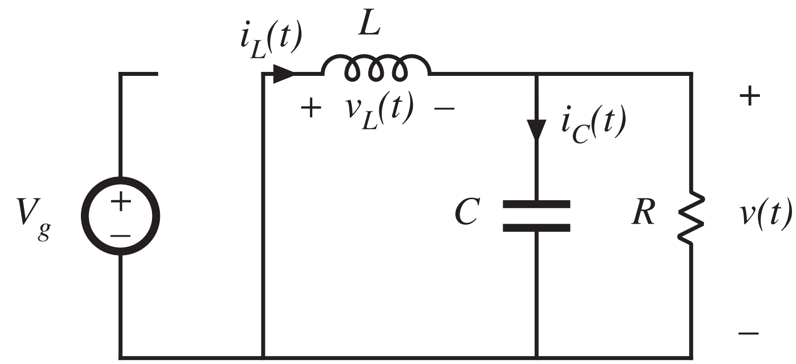
$$v_L(t) \approx V_g - V$$
$$i_C(t) \approx i_L(t) - V / R$$

Subinterval 2

$$v_L(t) = -\,v(t)$$
$$i_C(t) = i_L(t) - v(t)\,/\,R$$

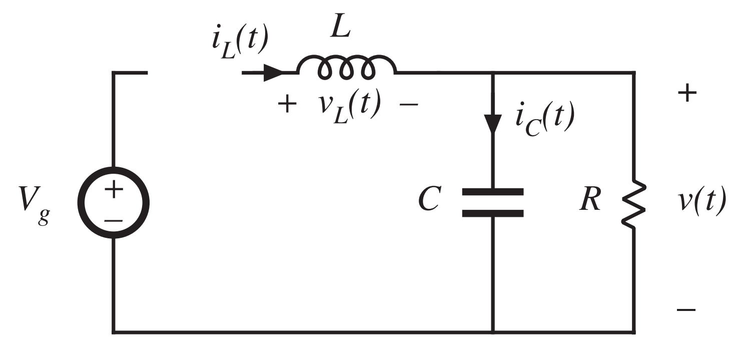
Small ripple approximation for $v(t)$ but not for $i(t)$:

$$v_L(t) \approx -\,V$$
$$i_C(t) \approx i_L(t) - V\,/\,R$$



Subinterval 3

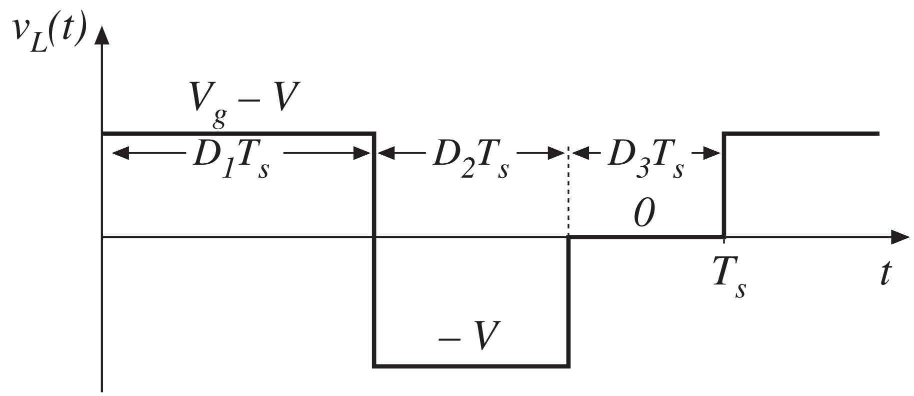
$$v_L = 0, \quad i_L = 0$$
$$i_C(t) = i_L(t) - v(t) / R$$



Small ripple approximation:

$$v_L(t) = 0$$
$$i_C(t) = -V / R$$

Inductor volt-second balance



Volt-second balance:

$$\left\langle v_L(t) \right\rangle = D_1(V_g - V) + D_2(-V) + D_3(0) = 0$$

Solve for V:

$$V = V_g \frac{D_1}{D_1 + D_2}$$

note that D_2 is unknown

Capacitor charge balance

node equation:

$$i_L(t) = i_C(t) + V / R$$

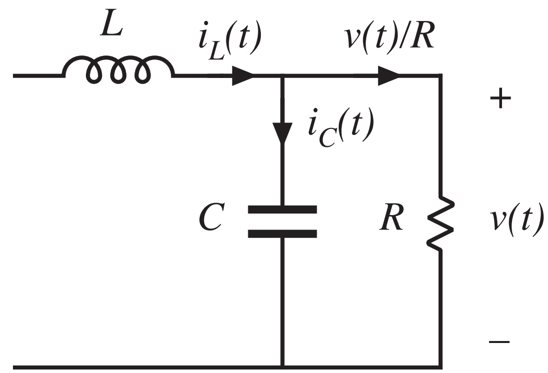
capacitor charge balance:

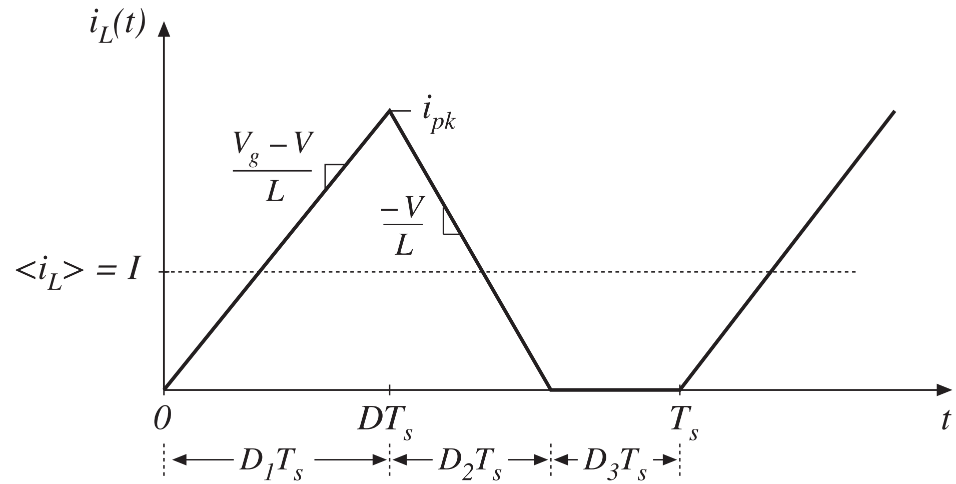
$$\langle i_C \rangle = 0$$

hence

$$\langle i_L \rangle = V / R$$

must compute dc component of inductor current and equate to load current (for this buck converter example)





Inductor current waveform

peak current:

$$i_L(D_1T_s) = i_{pk} = \frac{V_g - V}{L}\, D_1T_s$$

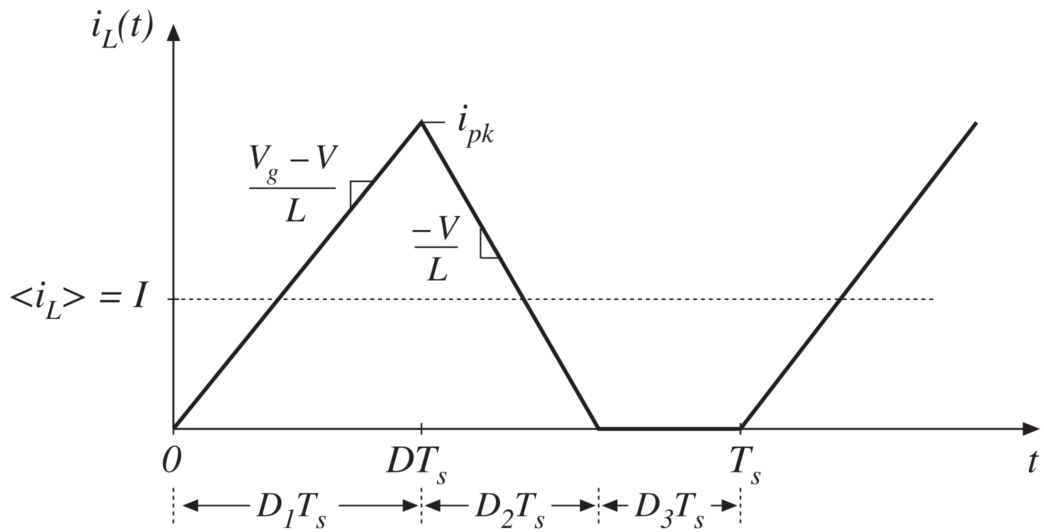
average current:

$$\left\langle i_L \right\rangle = \frac{1}{T_s}\int_0^{T_s} i_L(t)\, dt$$

triangle area formula:

$$\int_0^{T_s} i_L(t)\, dt = \frac{1}{2}\, i_{pk}\,(D_1 + D_2)T_s$$

$$\left\langle i_L \right\rangle = (V_g - V)\,\frac{D_1T_s}{2L}\,(D_1 + D_2)$$



equate dc component to dc load current:

$$\frac{V}{R} = \frac{D_1T_s}{2L}\,(D_1 + D_2)\,(V_g - V)$$

Solution for V

Two equations and two unknowns (V and D_2):

$$V = V_g \frac{D_1}{D_1 + D_2}$$ (from inductor volt-second balance)

$$\frac{V}{R} = \frac{D_1 T_s}{2L} (D_1 + D_2)(V_g - V)$$ (from capacitor charge balance)

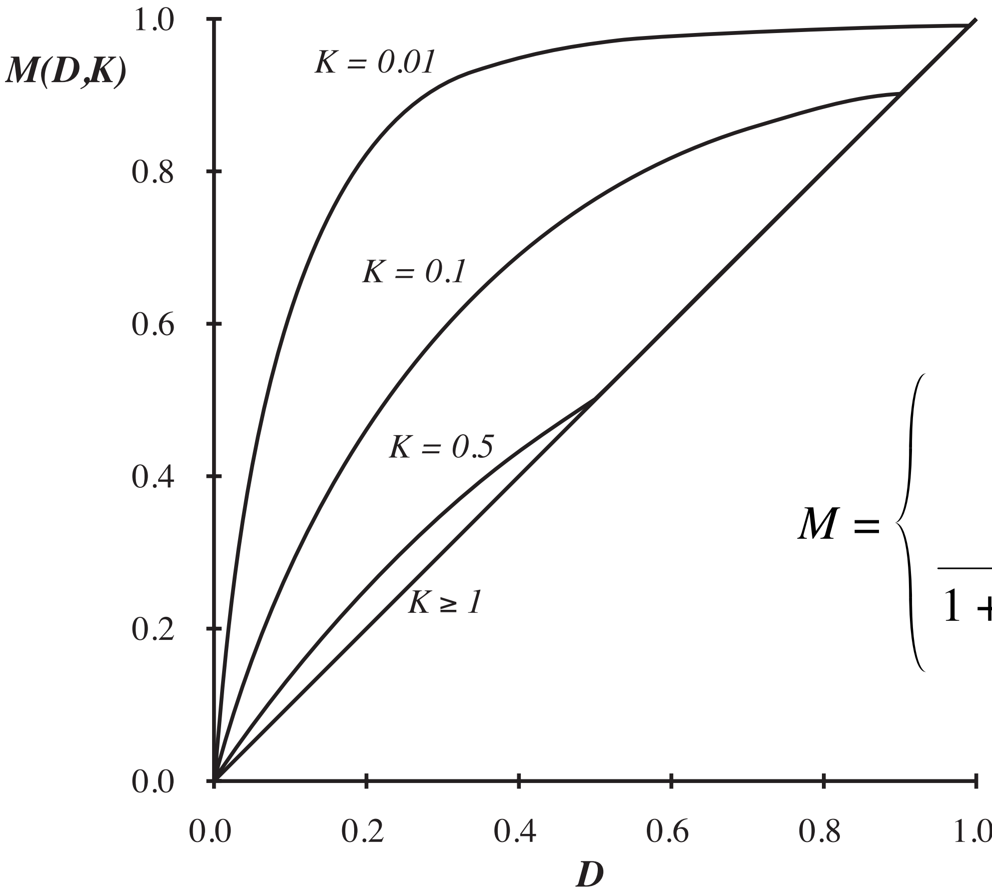
Eliminate D_2 , solve for V :

$$\frac{V}{V_g} = \frac{2}{1 + \sqrt{1 + 4K / D_1^2}}$$

where $K = 2L / RT_s$

valid for $K < K_{crit}$

Buck converter *M(D,K)*



$$M = \begin{cases} D & \text{for } K > K_{crit} \\ \dfrac{2}{1 + \sqrt{1 + 4K / D^2}} & \text{for } K < K_{crit} \end{cases}$$