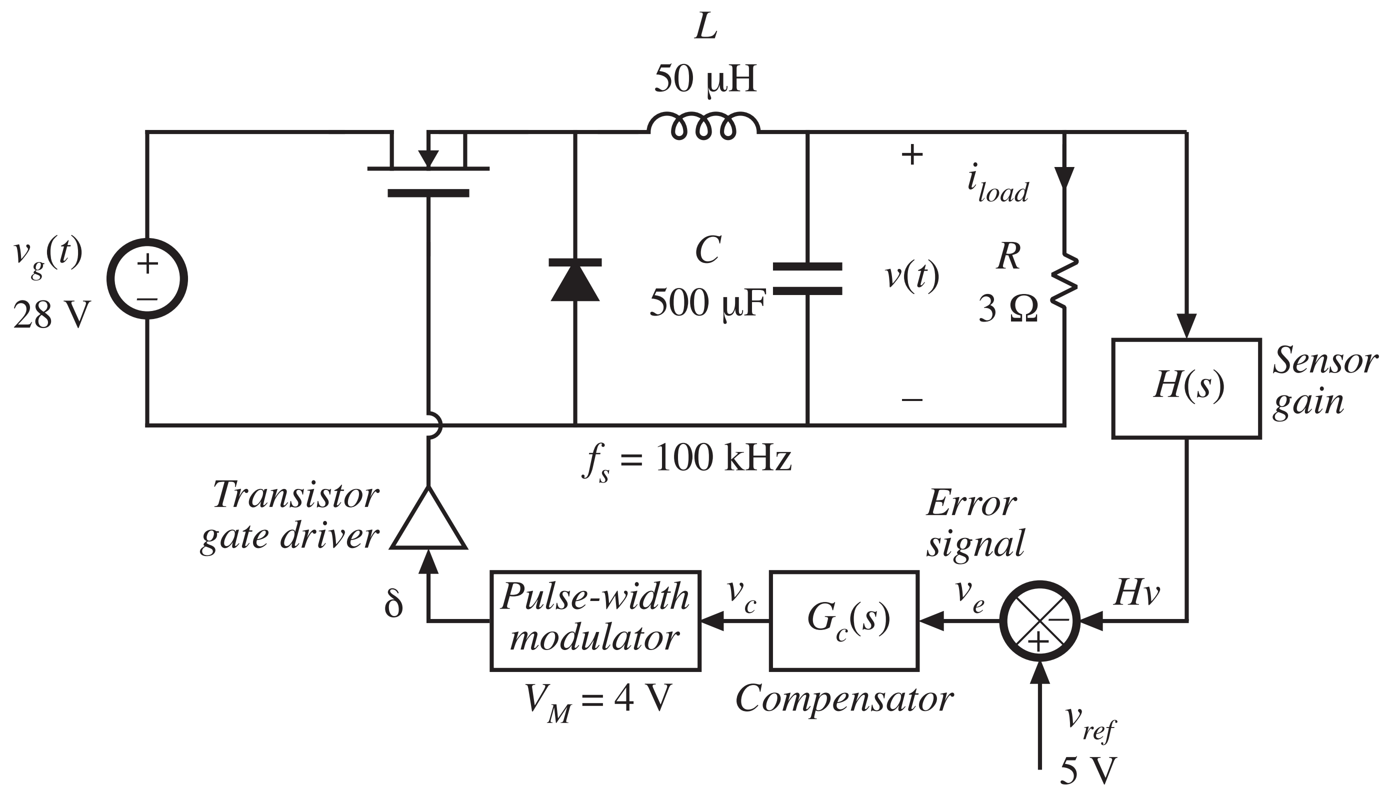
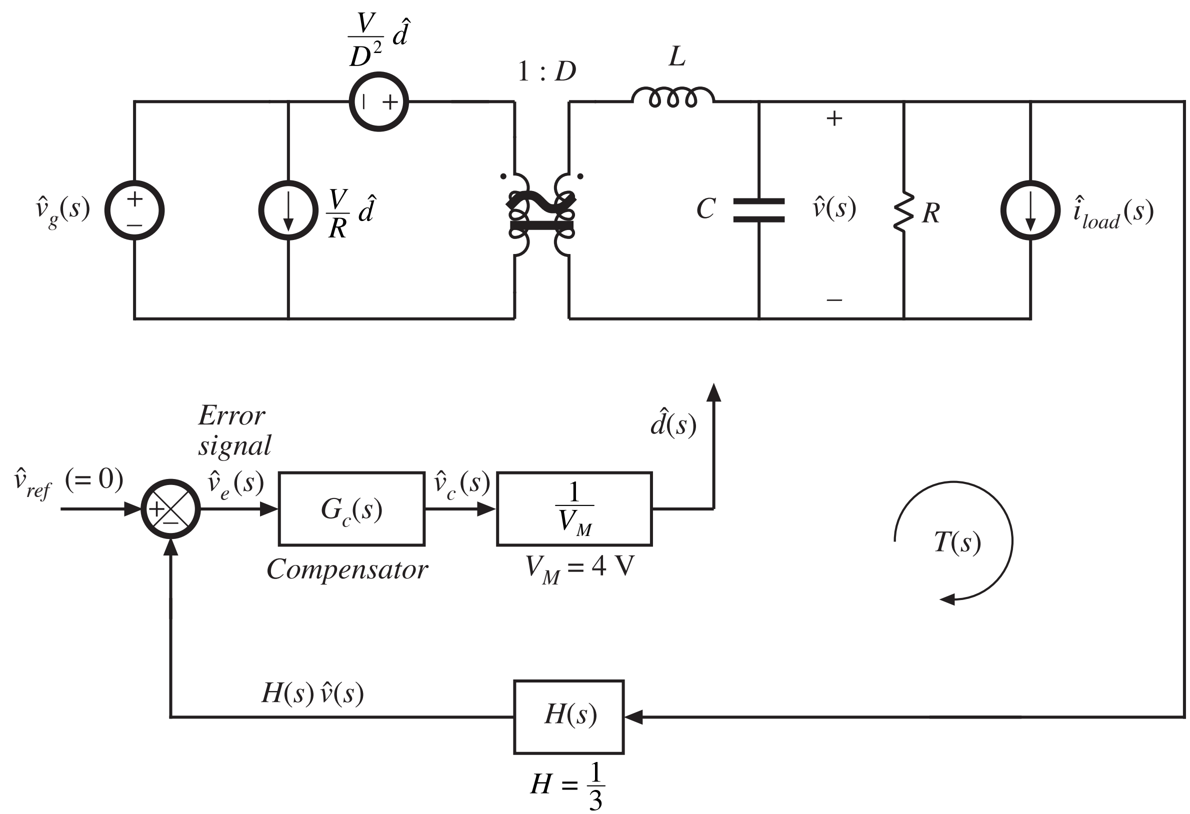
9.5.4. Design example



Quiescent operating point

Input voltage	$V_g = 28\text{V}$
Output	$V = 15\text{V},\ I_{load} = 5\text{A},\ R = 3\Omega$
Quiescent duty cycle	$D = 15/28 = 0.536$
Reference voltage	$V_{ref} = 5\text{V}$
Quiescent value of control voltage	$V_c = DV_M = 2.14\text{V}$
Gain $H(s)$	$H = V_{ref}/V = 5/15 = 1/3$

Small-signal model



Open-loop control-to-output transfer function $G_{vd}(s)$

$$G_{vd}(s) = \frac{V}{D} \frac{1}{1 + s\frac{L}{R} + s^2 LC}$$

standard form:

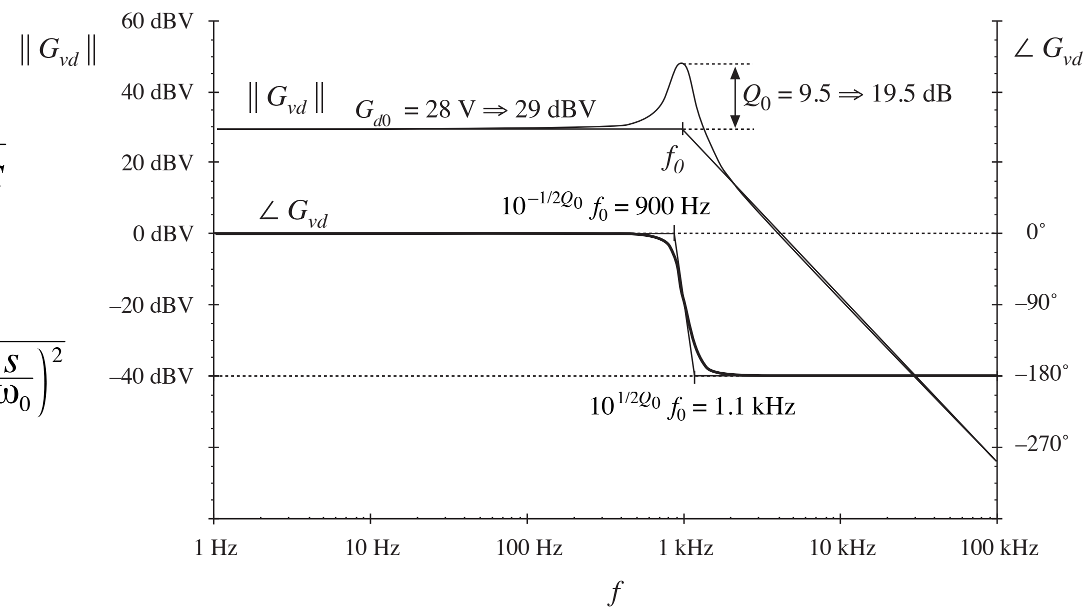
$$G_{vd}(s) = G_{d0} \frac{1}{1 + \frac{s}{Q_0 \omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

salient features:

$$G_{d0} = \frac{V}{D} = 28\text{V}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = 1\text{kHz}$$

$$Q_0 = R\sqrt{\frac{C}{L}} = 9.5 \Rightarrow 19.5\text{dB}$$



Open-loop line-to-output transfer function and output impedance

$$G_{vg}(s) = D \frac{1}{1 + s\frac{L}{R} + s^2 LC}$$

—same poles as control-to-output transfer function
standard form:

$$G_{vg}(s) = G_{g0} \frac{1}{1 + \frac{s}{Q_0 \omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

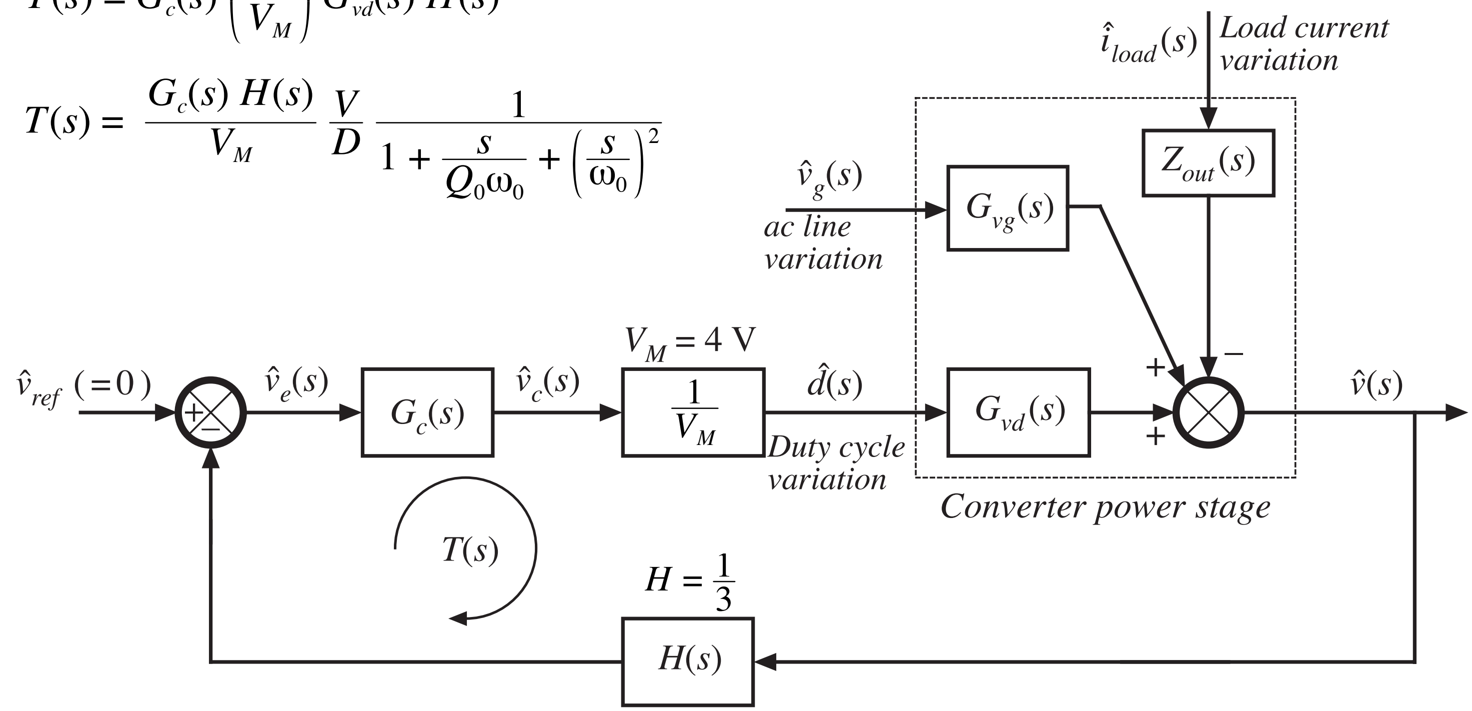
Output impedance:

$$Z_{out}(s) = R \parallel \frac{1}{sC} \parallel sL = \frac{sL}{1 + s\frac{L}{R} + s^2 LC}$$

System block diagram

$$T(s) = G_c(s)\left(\frac{1}{V_M}\right) G_{vd}(s)\, H(s)$$

$$T(s) = \frac{G_c(s)\, H(s)}{V_M}\, \frac{V}{D}\, \frac{1}{1 + \frac{s}{Q_0\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

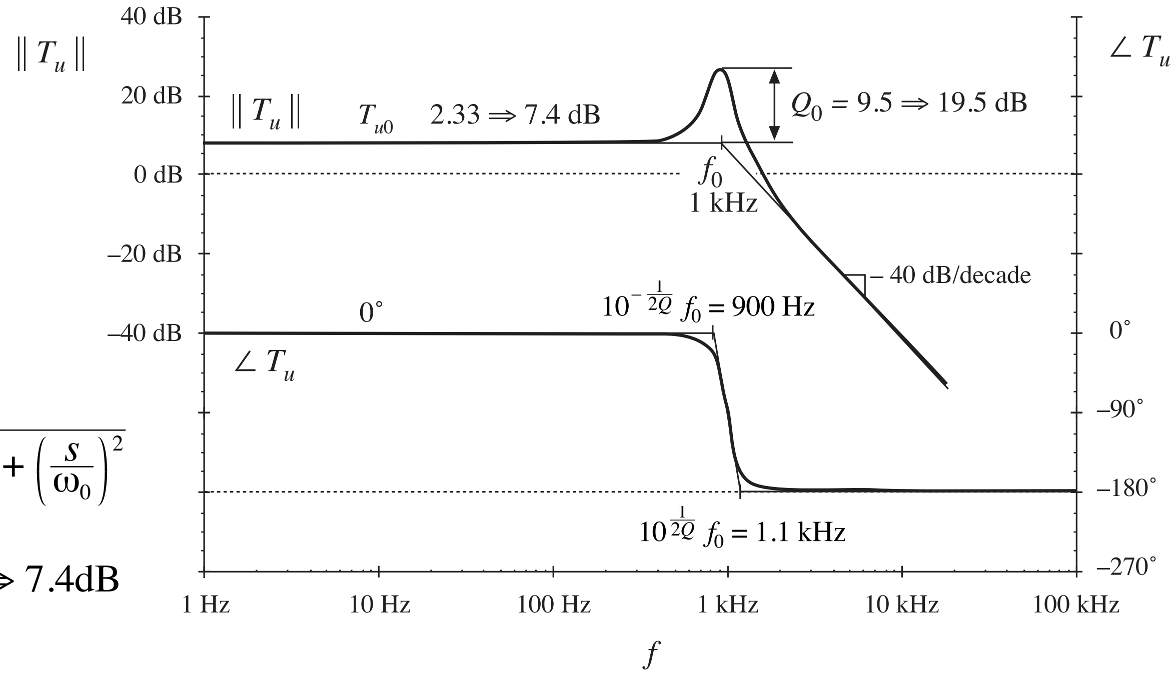


Uncompensated loop gain (with $G_c = 1$)

With $G_c = 1$, the loop gain is

$$T_u(s) = T_{u0} \frac{1}{1 + \frac{s}{Q_0 \omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

$$T_{u0} = \frac{H\ V}{D\ V_M} = 2.33 \Rightarrow 7.4\text{dB}$$



$f_c = 1.8$ kHz, $\varphi_m = 5^\circ$

Lead compensator design

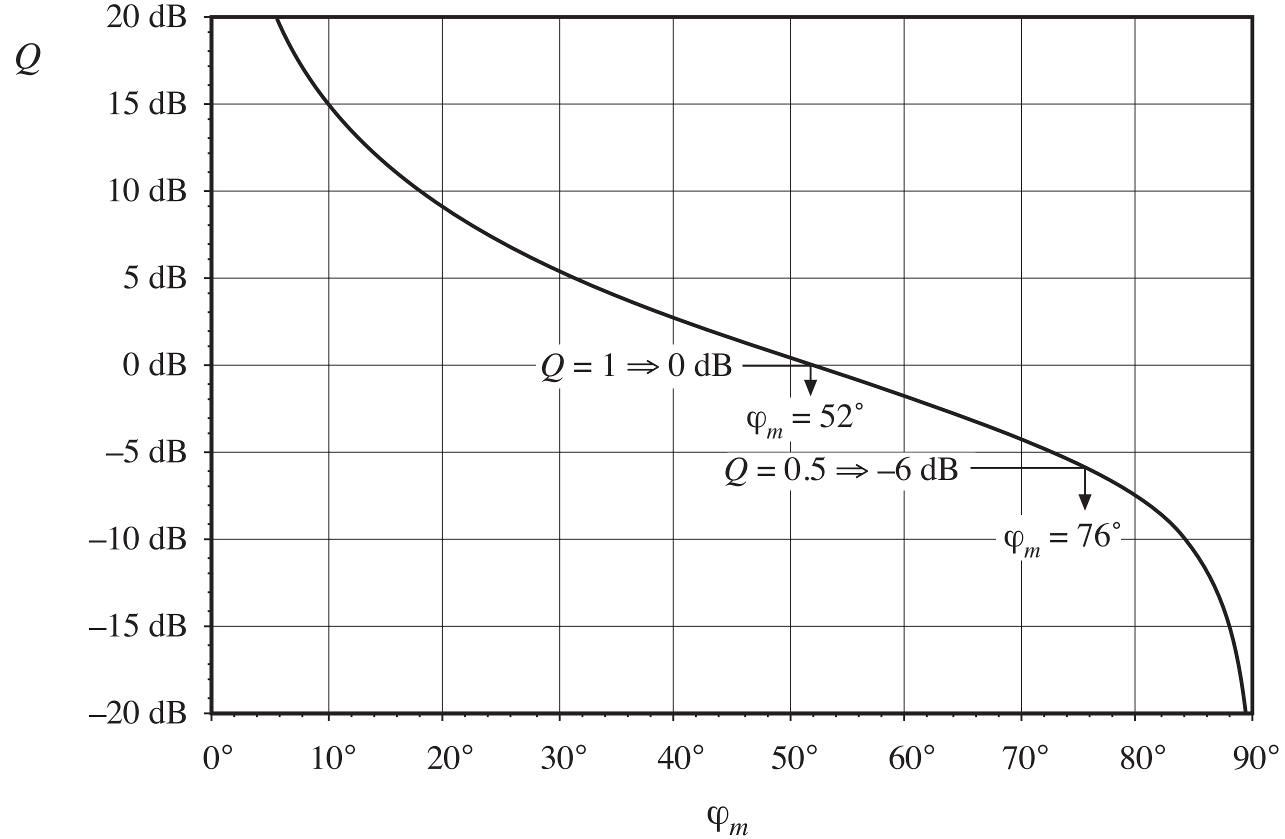
- Obtain a crossover frequency of 5 kHz, with phase margin of 52°
- T_u has phase of approximately – 180° at 5 kHz, hence lead (PD) compensator is needed to increase phase margin.
- Lead compensator should have phase of + 52° at 5 kHz
- T_u has magnitude of – 20.6 dB at 5 kHz
- Lead compensator gain should have magnitude of + 20.6 dB at 5 kHz
- Lead compensator pole and zero frequencies should be

$$f_z = (5\text{kHz})\sqrt{\frac{1 - \sin(52°)}{1 + \sin(52°)}} = 1.7\text{kHz}$$

$$f_p = (5\text{kHz})\sqrt{\frac{1 + \sin(52°)}{1 - \sin(52°)}} = 14.5\text{kHz}$$

- Compensator dc gain should be $G_{c0} = \left(\frac{f_c}{f_0}\right)^2 \frac{1}{T_{u0}} \sqrt{\frac{f_z}{f_p}} = 3.7 \Rightarrow 11.3\text{dB}$

Q vs. φ_m

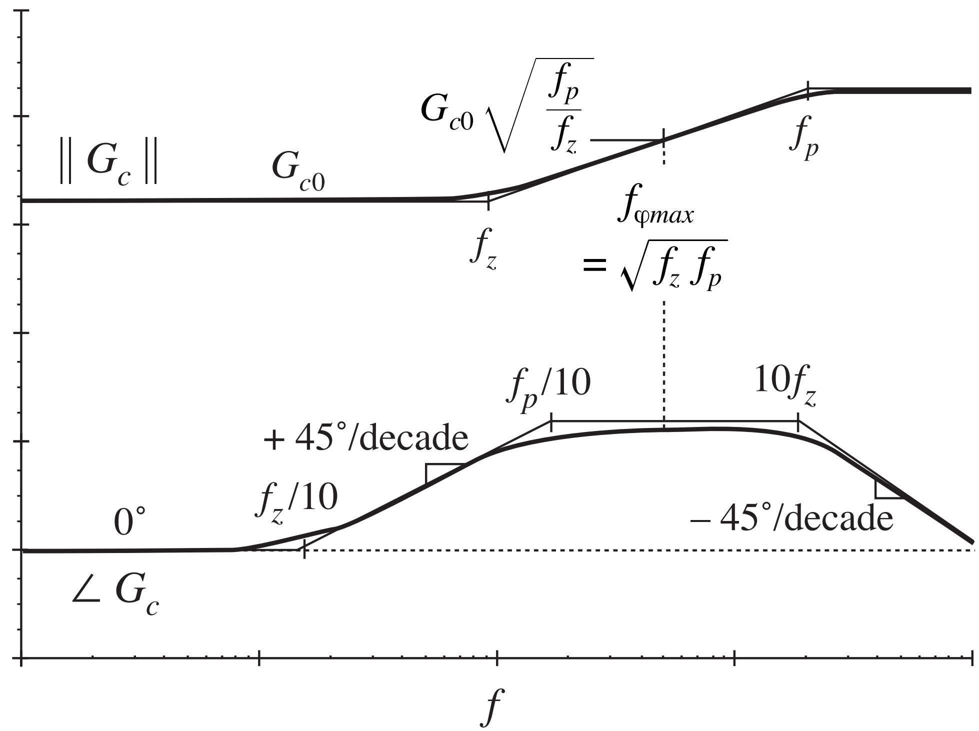


Q
20 dB
15 dB
10 dB
5 dB
0 dB
–5 dB
–10 dB
–15 dB
–20 dB
Q = 1 ⇒ 0 dB
φm = 52˚
Q = 0.5 ⇒ –6 dB
φm = 76˚
0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
φm

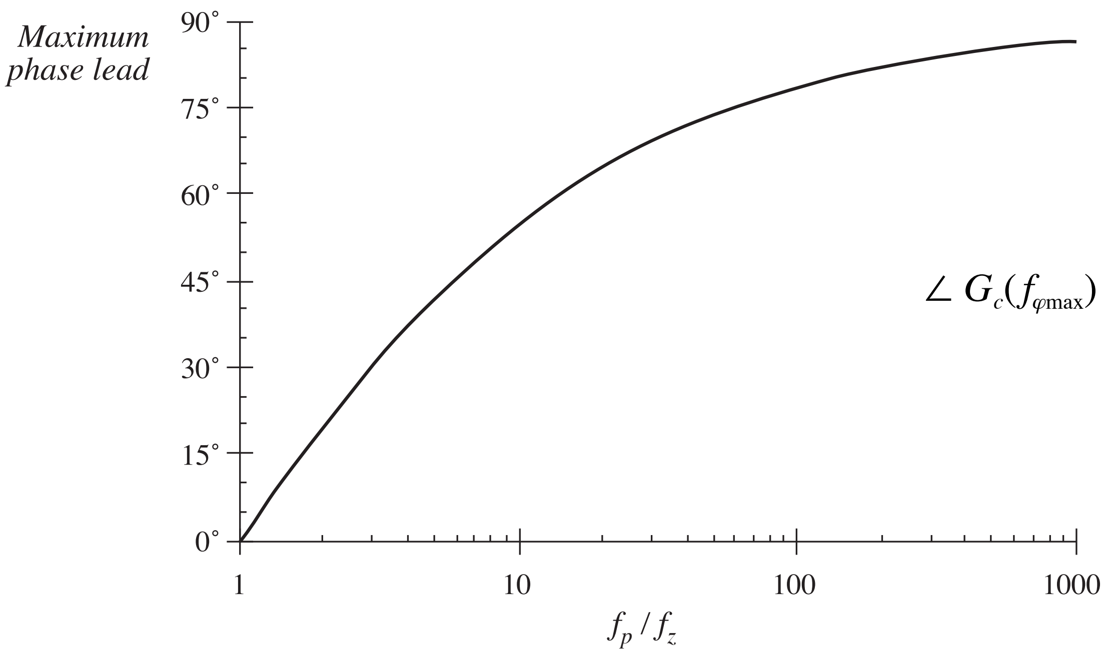
9.5.1. Lead (PD) compensator

$$G_c(s) = G_{c0} \frac{\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_p}\right)}$$

Improves phase margin



Lead compensator: maximum phase lead



$$f_{\varphi max} = \sqrt{f_z f_p}$$

$$\angle\, G_c(f_{\varphi max}) = \tan^{-1}\left(\frac{\sqrt{\frac{f_p}{f_z}} - \sqrt{\frac{f_z}{f_p}}}{2}\right)$$

$$\frac{f_p}{f_z} = \frac{1 + \sin(\theta)}{1 - \sin(\theta)}$$

Lead compensator design

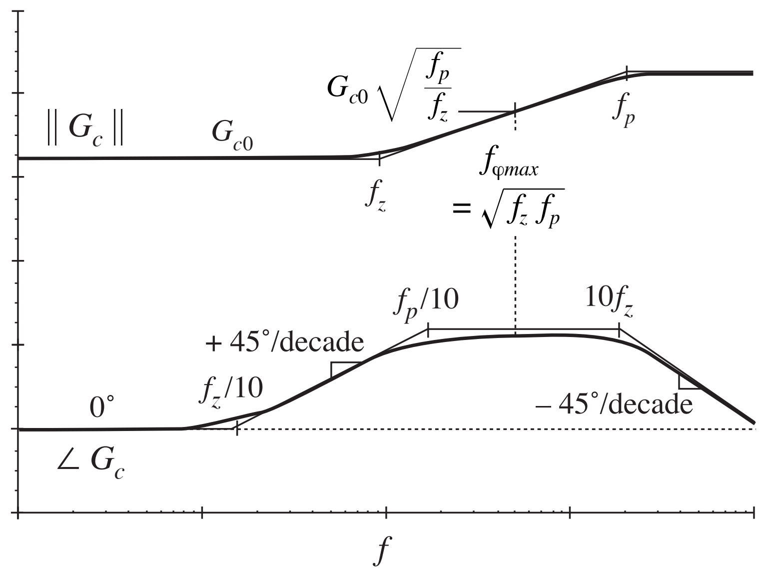
To optimally obtain a compensator phase lead of θ at frequency f_c, the pole and zero frequencies should be chosen as follows:

$$f_z = f_c \sqrt{\frac{1 - \sin(\theta)}{1 + \sin(\theta)}}$$

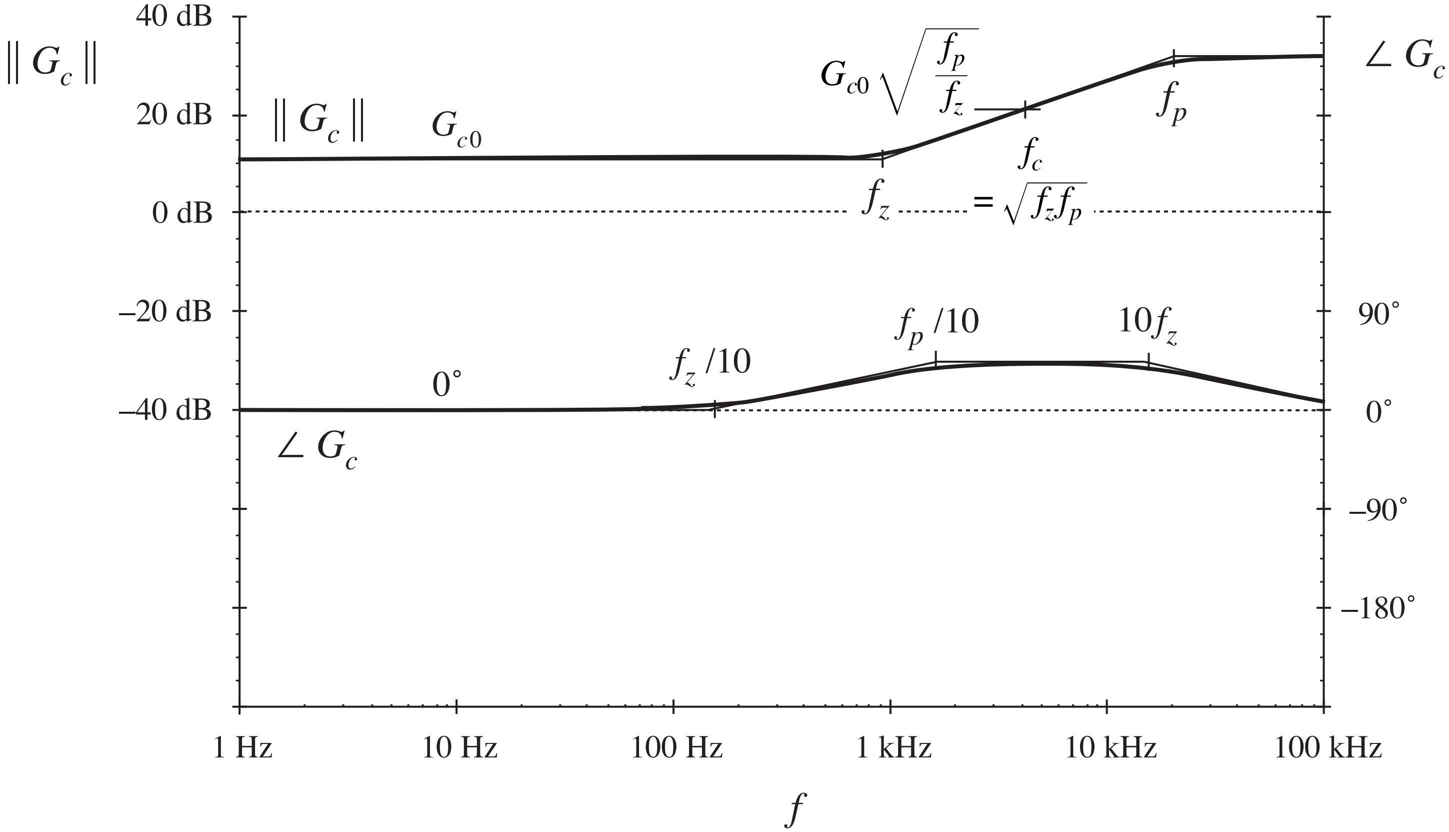
$$f_p = f_c \sqrt{\frac{1 + \sin(\theta)}{1 - \sin(\theta)}}$$

If it is desired that the magnitude of the compensator gain at f_c be unity, then G_{c0} should be chosen as

$$G_{c0} = \sqrt{\frac{f_z}{f_p}}$$

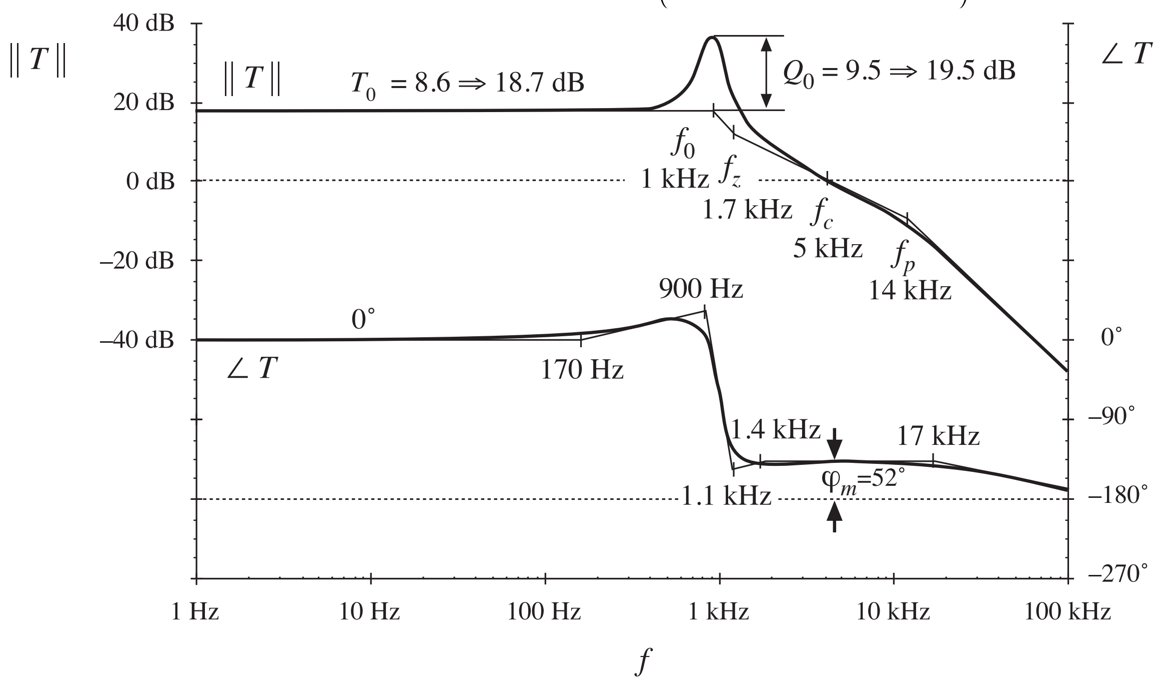


Lead compensator Bode plot

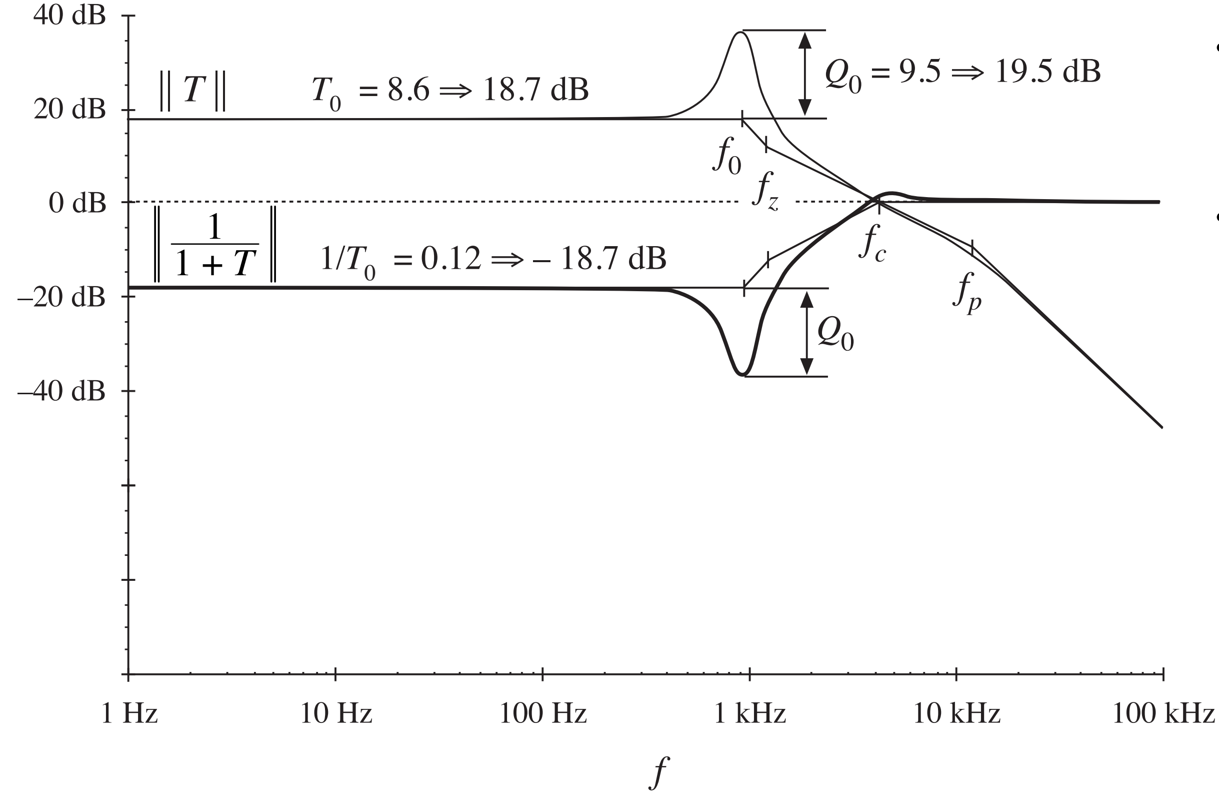


Loop gain, with lead compensator

$$T(s) = T_{u0}\, G_{c0} \frac{\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_p}\right)\left(1 + \frac{s}{Q_0\omega_0} + \left(\frac{s}{\omega_0}\right)^2\right)}$$



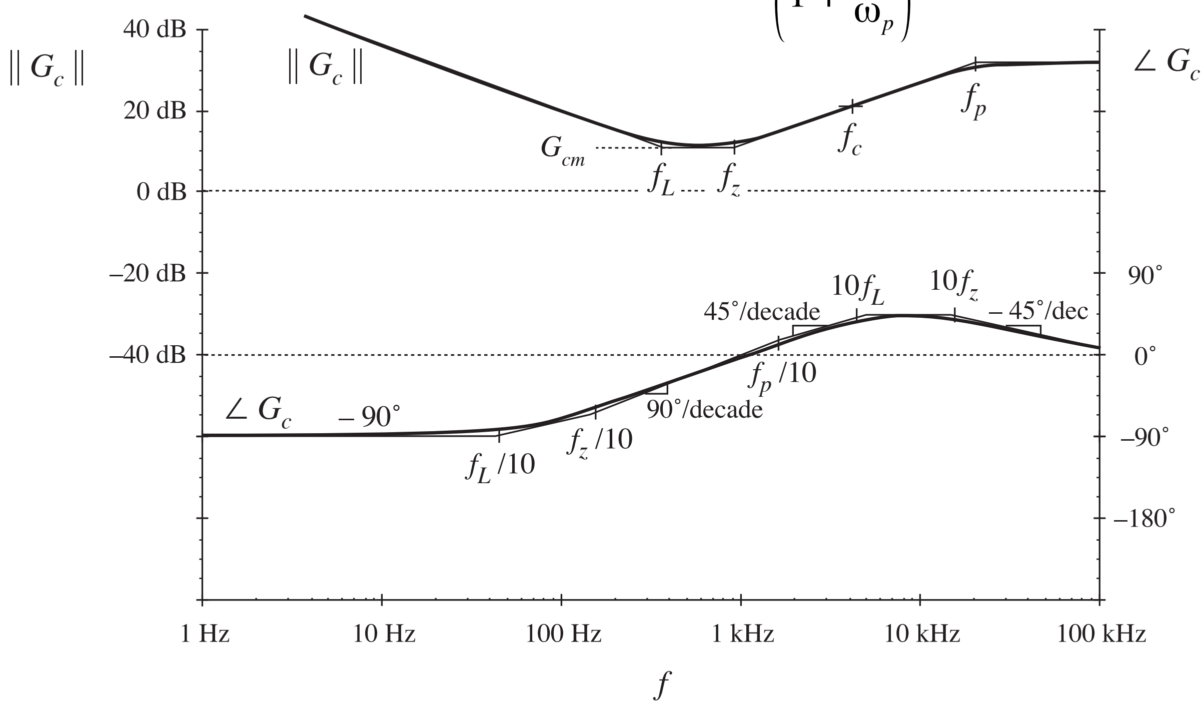
1/(1+T), with lead compensator



- need more low-frequency loop gain
- hence, add inverted zero (PID controller)

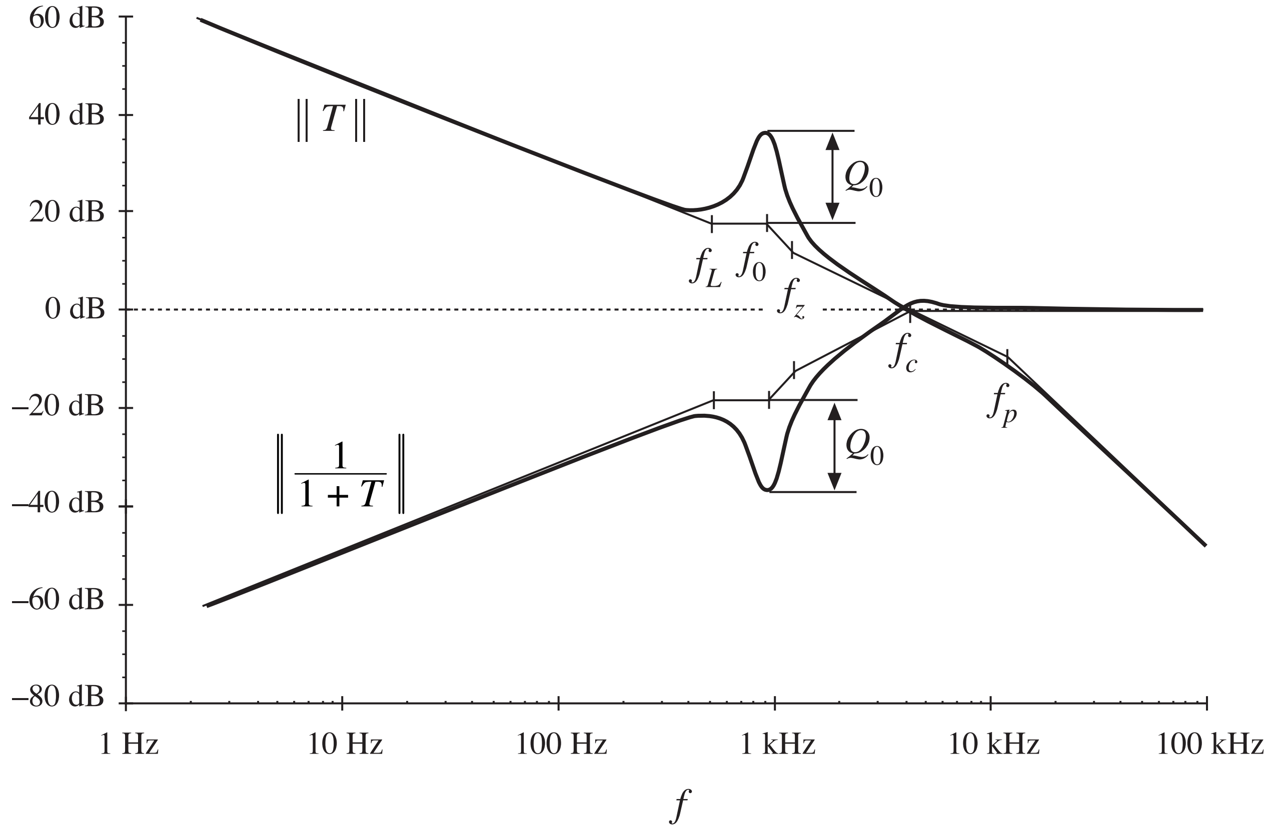
Improved compensator (PID)

$$G_c(s) = G_{cm} \frac{\left(1 + \frac{s}{\omega_z}\right)\left(1 + \frac{\omega_L}{s}\right)}{\left(1 + \frac{s}{\omega_p}\right)}$$



- add inverted zero to PD compensator, without changing dc gain or corner frequencies
- choose f_L to be $f_c/10$, so that phase margin is unchanged

$T(s)$ and $1/(1+T(s))$, with PID compensator



Open-loop line-to-output transfer function and output impedance

$$G_{vg}(s) = D \frac{1}{1 + s\frac{L}{R} + s^2 LC}$$

—same poles as control-to-output transfer function
standard form:

$$G_{vg}(s) = G_{g0} \frac{1}{1 + \frac{s}{Q_0 \omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

Output impedance:

$$Z_{out}(s) = R \parallel \frac{1}{sC} \parallel sL = \frac{sL}{1 + s\frac{L}{R} + s^2 LC}$$

Line-to-output transfer function