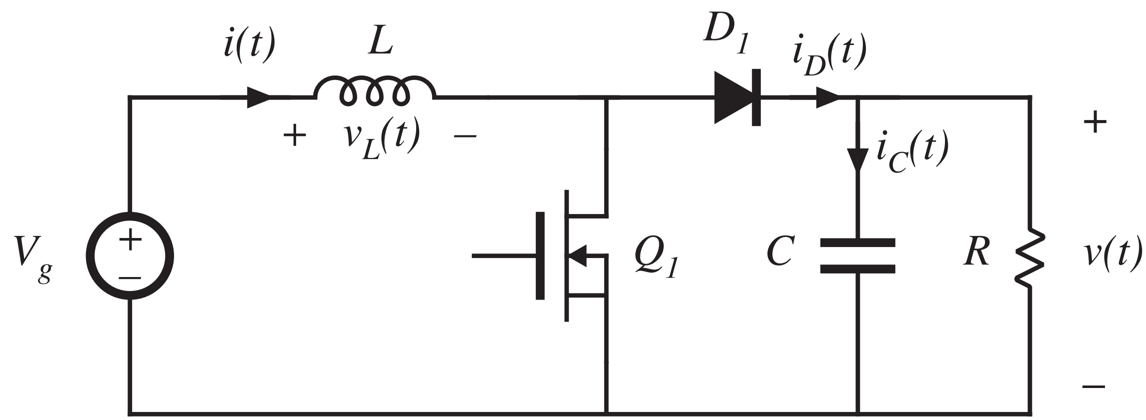
5.3. Boost converter example



Mode boundary:

$$I > \Delta i_L \quad \text{for CCM}$$
$$I < \Delta i_L \quad \text{for DCM}$$

Previous CCM soln:

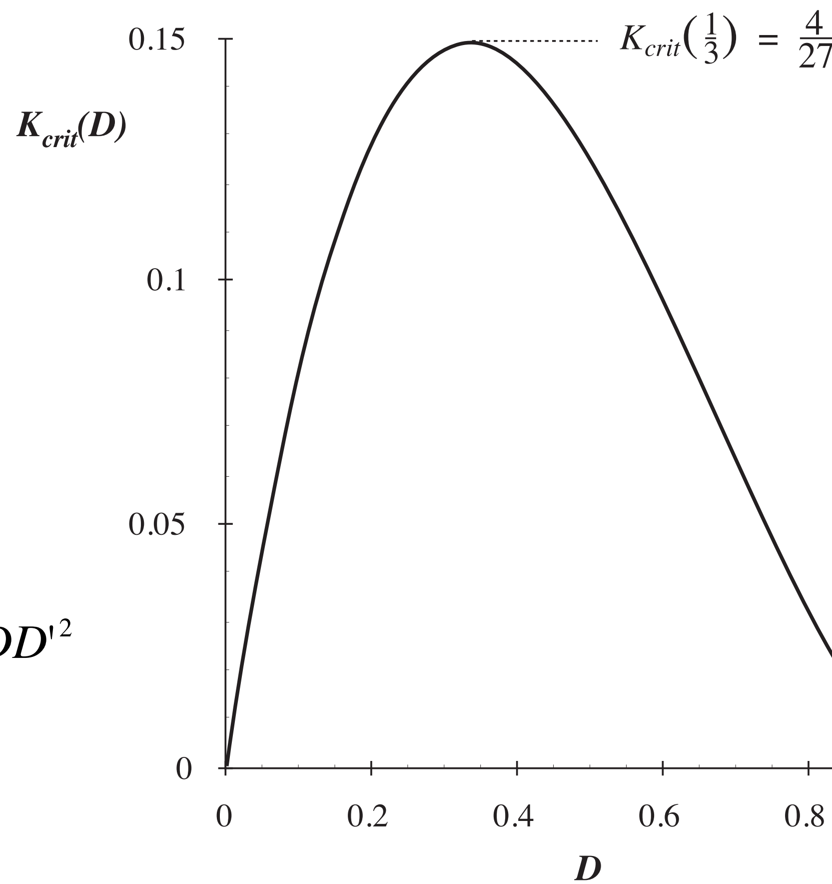
$$I = \frac{V_g}{D'^2 R} \qquad \Delta i_L = \frac{V_g}{2L} D T_s$$

Mode boundary

$$\frac{V_g}{D'^2R} > \frac{DT_sV_g}{2L} \qquad \textit{for CCM}$$

$$\frac{2L}{RT_s} > DD'^2 \qquad \textit{for CCM}$$

$$K > K_{crit}(D) \qquad \textit{for CCM}$$
$$K < K_{crit}(D) \qquad \textit{for DCM}$$
$$\textit{where} \quad K = \frac{2L}{RT_s} \quad \textit{and} \quad K_{crit}(D) = DD'^2$$

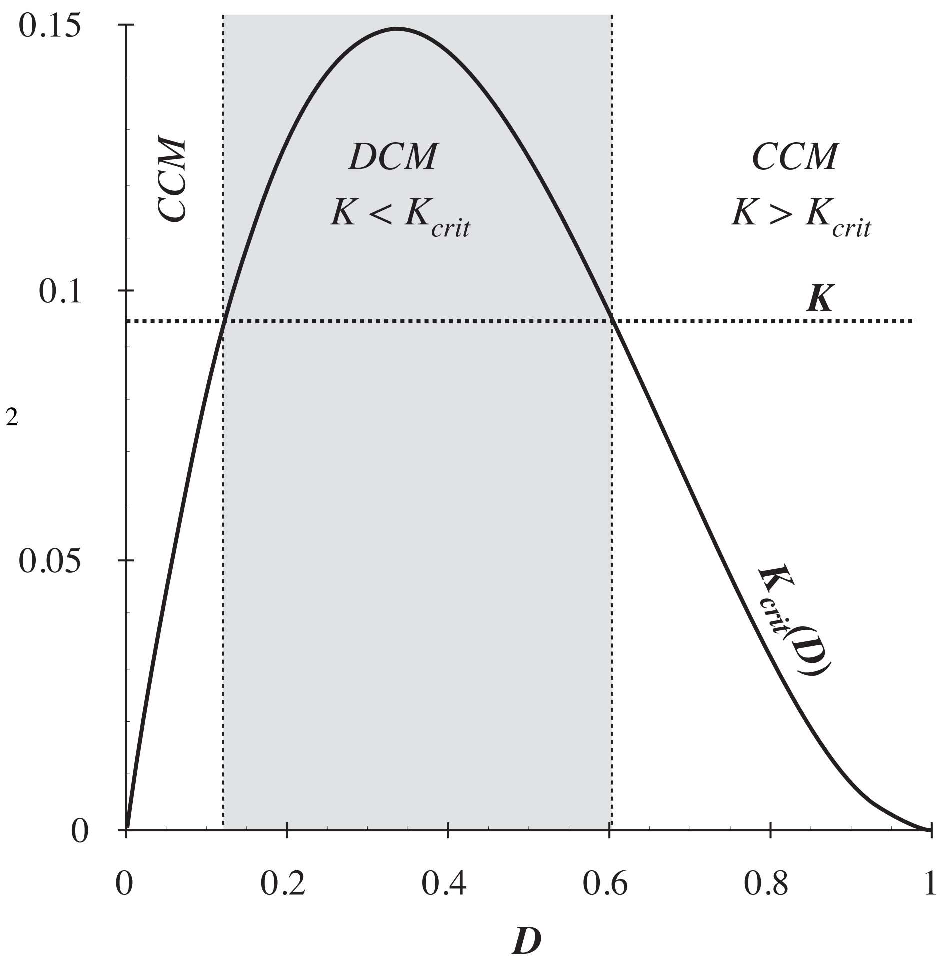


Mode boundary

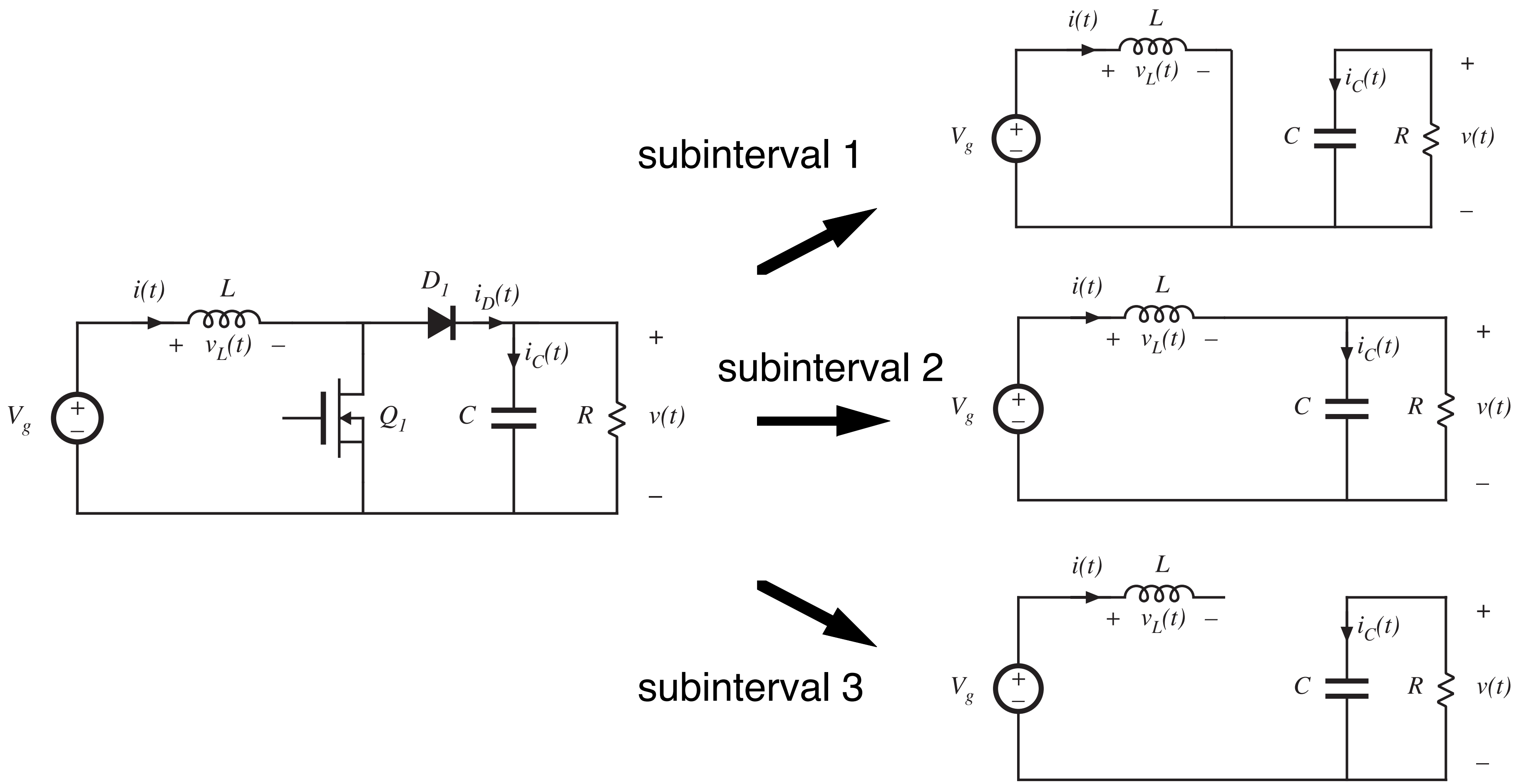
$$K > K_{crit}(D) \qquad \textit{for CCM}$$

$$K < K_{crit}(D) \qquad \textit{for DCM}$$

$$\textit{where} \quad K = \frac{2L}{RT_s} \quad \textit{and} \quad K_{crit}(D) = DD'^2$$



Conversion ratio: DCM boost

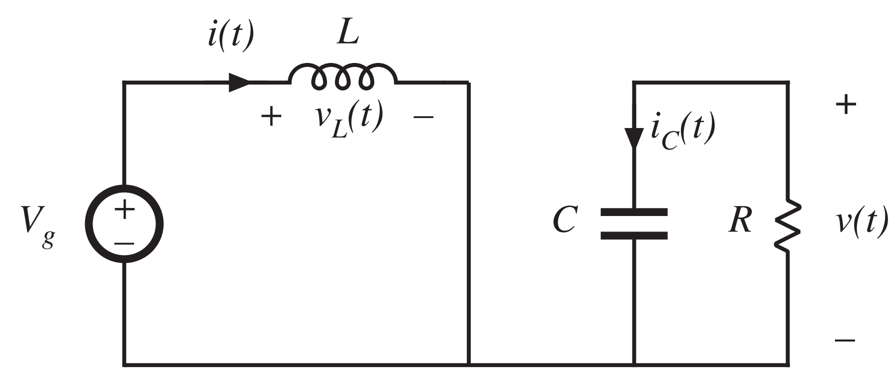


Subinterval 1

$$v_L(t) = V_g$$
$$i_C(t) = -\, v(t) \,/\, R$$

Small ripple approximation for *v(t)* (but not for *i(t)*!):

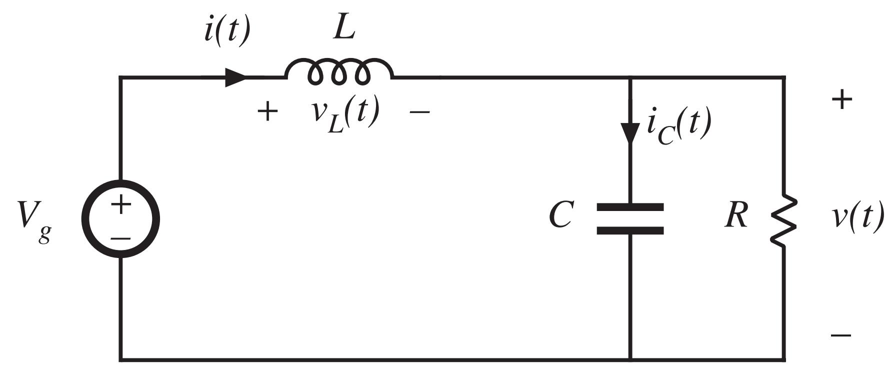
$$v_L(t) \approx V_g$$
$$i_C(t) \approx -\, V \,/\, R$$



$$0 < t < D_1 T_s$$

Subinterval 2

$$v_L(t) = V_g - v(t)$$
$$i_C(t) = i(t) - v(t) / R$$



Small ripple approximation for $v(t)$ but not for $i(t)$:

$$v_L(t) \approx V_g - V$$
$$i_C(t) \approx i(t) - V / R$$

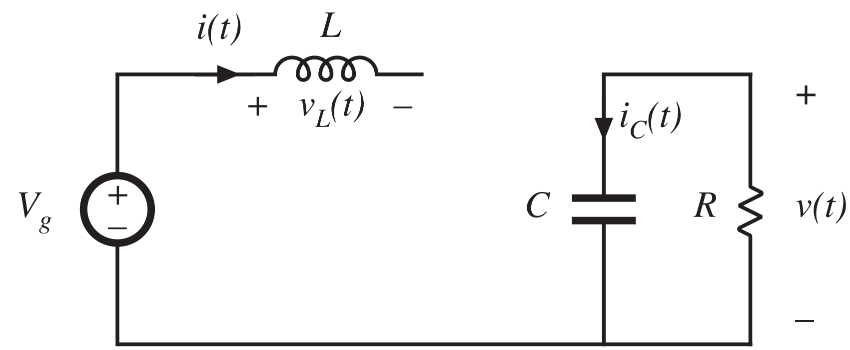
$$D_1 T_s < t < (D_1 + D_2) T_s$$

Subinterval 3

$$v_L = 0, \quad i = 0$$
$$i_C(t) = -\, v(t) \,/\, R$$

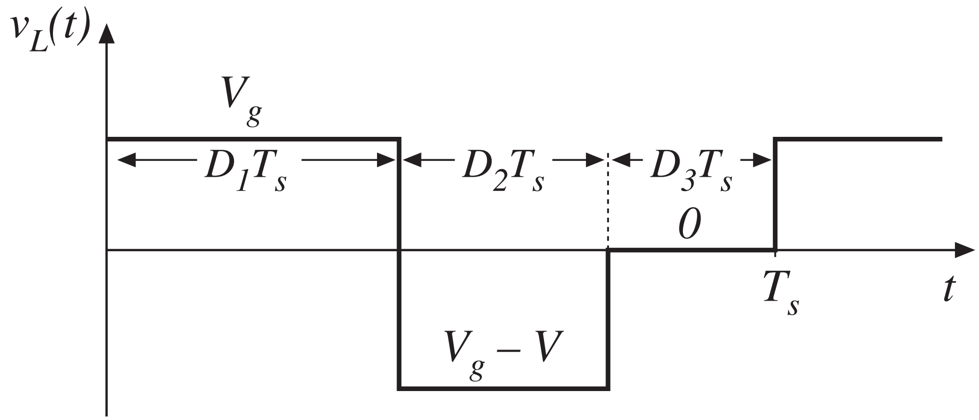
Small ripple approximation:

$$v_L(t) = 0$$
$$i_C(t) = -\, V \,/\, R$$



$$(D_1 + D_2)T_s < t < T_s$$

Inductor volt-second balance



Volt-second balance:

$$D_1V_g + D_2(V_g - V) + D_3(0) = 0$$

Solve for V:

$$V = \frac{D_1 + D_2}{D_2} V_g$$

note that D_2 is unknown

Capacitor charge balance

node equation:

$$i_D(t) = i_C(t) + v(t) / R$$

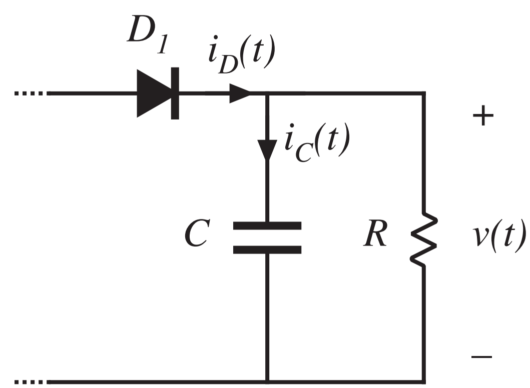
capacitor charge balance:

$$\left\langle i_C \right\rangle = 0$$

hence

$$\left\langle i_D \right\rangle = V / R$$

must compute dc component of diode current and equate to load current (for this boost converter example)



Inductor and diode current waveforms

peak current:

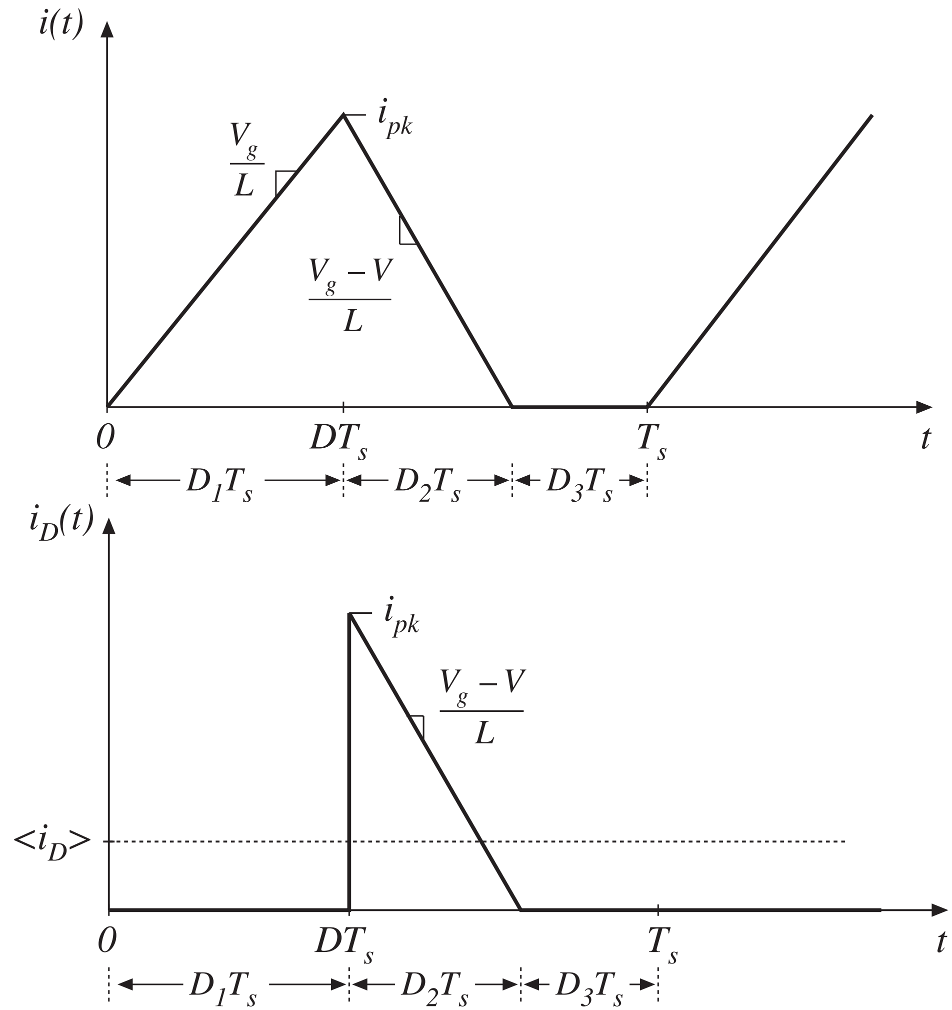
$$i_{pk} = \frac{V_g}{L} D_1 T_s$$

average diode current:

$$\left\langle i_D \right\rangle = \frac{1}{T_s} \int_0^{T_s} i_D(t)\, dt$$

triangle area formula:

$$\int_0^{T_s} i_D(t)\, dt = \frac{1}{2}\, i_{pk}\, D_2 T_s$$



Equate diode current to load current

average diode current:

$$\left\langle i_D \right\rangle = \frac{1}{T_s}\left(\frac{1}{2}\, i_{pk}\, D_2 T_s\right) = \frac{V_g D_1 D_2 T_s}{2L}$$

equate to dc load current:

$$\frac{V_g D_1 D_2 T_s}{2L} = \frac{V}{R}$$

Solution for V

Two equations and two unknowns (V and D_2):

$$V = \frac{D_1 + D_2}{D_2} V_g \qquad \text{(from inductor volt-second balance)}$$

$$\frac{V_g D_1 D_2 T_s}{2L} = \frac{V}{R} \qquad \text{(from capacitor charge balance)}$$

Eliminate D_2 , solve for V. From volt-sec balance eqn:

$$D_2 = D_1 \frac{V_g}{V - V_g}$$

Substitute into charge balance eqn, rearrange terms:

$$V^2 - VV_g - \frac{V_g^2 D_1^2}{K} = 0$$

Solution for V

$$V^2 - VV_g - \frac{V_g^2 D_1^2}{K} = 0$$

Use quadratic formula:

$$\frac{V}{V_g} = \frac{1 \pm \sqrt{1 + 4D_1^2 / K}}{2}$$

Note that one root leads to positive V, while other leads to negative V. Select positive root:

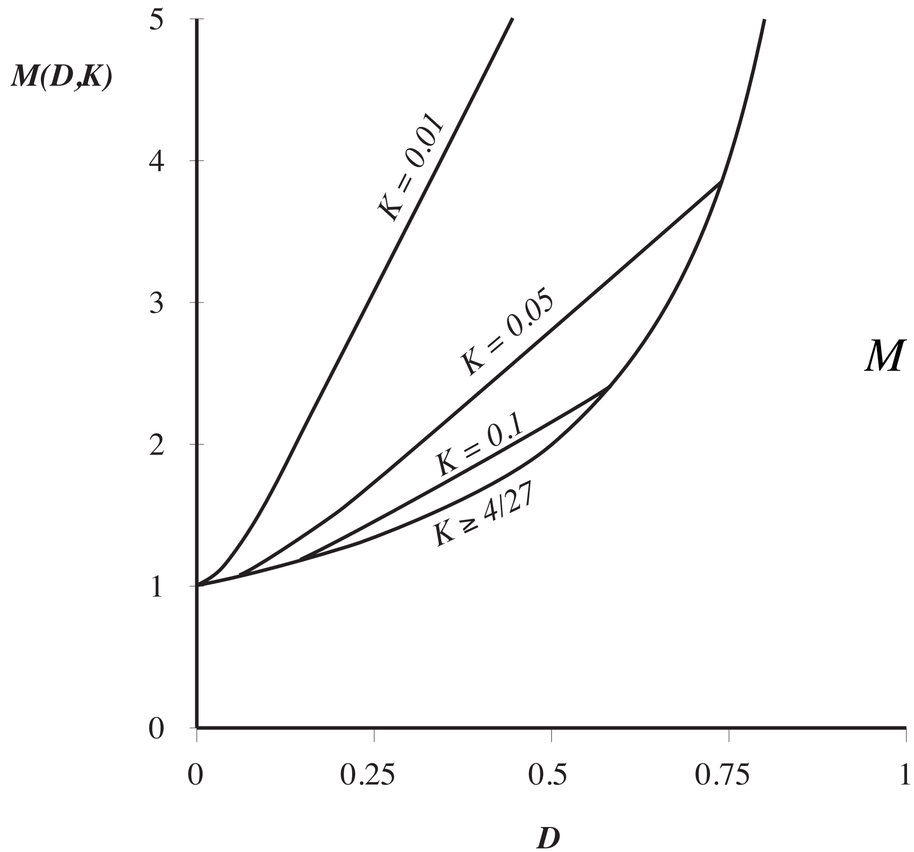
$$\frac{V}{V_g} = M(D_1, K) = \frac{1 + \sqrt{1 + 4D_1^2 / K}}{2}$$

where $K = 2L / RT_s$

valid for $K < K_{crit}(D)$

Transistor duty cycle D = interval *1* duty cycle D_1

Boost converter characteristics



$$M = \begin{cases} \dfrac{1}{1-D} & \text{for } K > K_{crit} \\ \dfrac{1+\sqrt{1+4D^2/K}}{2} & \text{for } K < K_{crit} \end{cases}$$

Approximate M in DCM:

$$M \approx \frac{1}{2} + \frac{D}{\sqrt{K}}$$

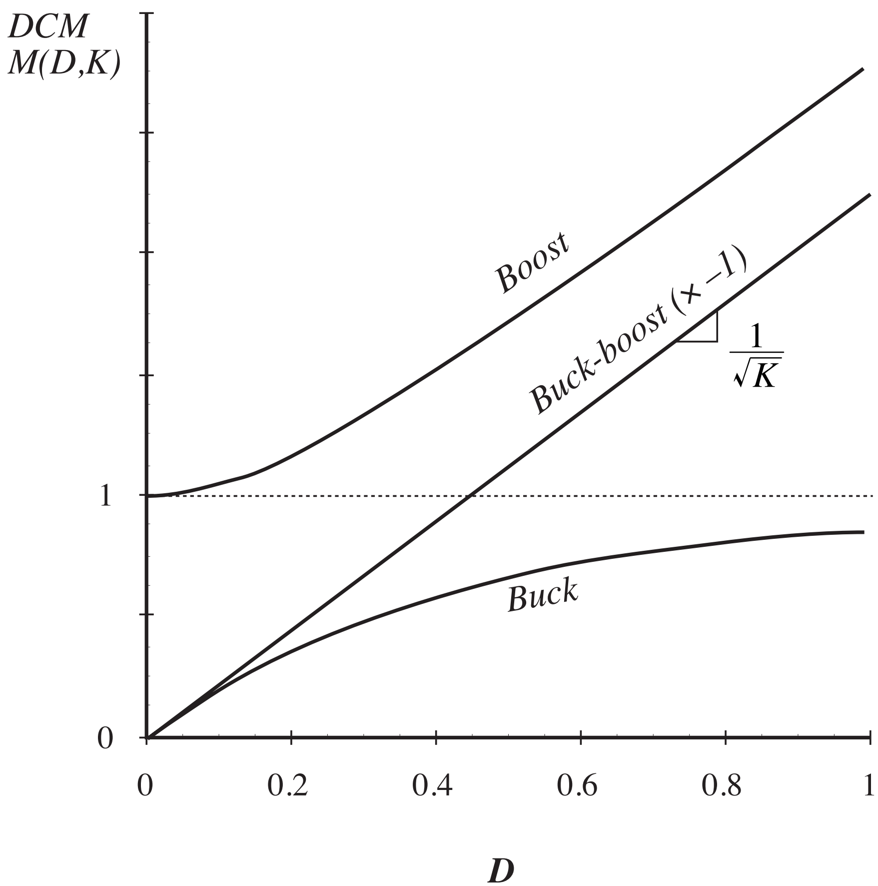
Summary of DCM characteristics

Table 5.2. Summary of CCM-DCM characteristics for the buck, boost, and buck-boost converters

Converter	$K_{crit}(D)$	*DCM* $M(D,K)$	*DCM* $D_2(D,K)$	*CCM* $M(D)$
Buck	$(1-D)$	$\dfrac{2}{1+\sqrt{1+4K/D^2}}$	$\dfrac{K}{D}M(D,K)$	D
Boost	$D(1-D)^2$	$\dfrac{1+\sqrt{1+4D^2/K}}{2}$	$\dfrac{K}{D}M(D,K)$	$\dfrac{1}{1-D}$
Buck-boost	$(1-D)^2$	$-\dfrac{D}{\sqrt{K}}$	$\sqrt{K}$	$-\dfrac{D}{1-D}$

with $K = 2L/RT_s$. *DCM occurs for* $K < K_{crit}$.

Summary of DCM characteristics



- DCM buck and boost characteristics are asymptotic to M = 1 and to the DCM buck-boost characteristic
- DCM buck-boost characteristic is linear
- CCM and DCM characteristics intersect at mode boundary. Actual M follows characteristic having larger magnitude
- DCM boost characteristic is nearly linear