7.5 The canonical circuit model

All PWM CCM dc-dc converters perform the same basic functions:

- Transformation of voltage and current levels, ideally with 100% efficiency
- Low-pass filtering of waveforms
- Control of waveforms by variation of duty cycle

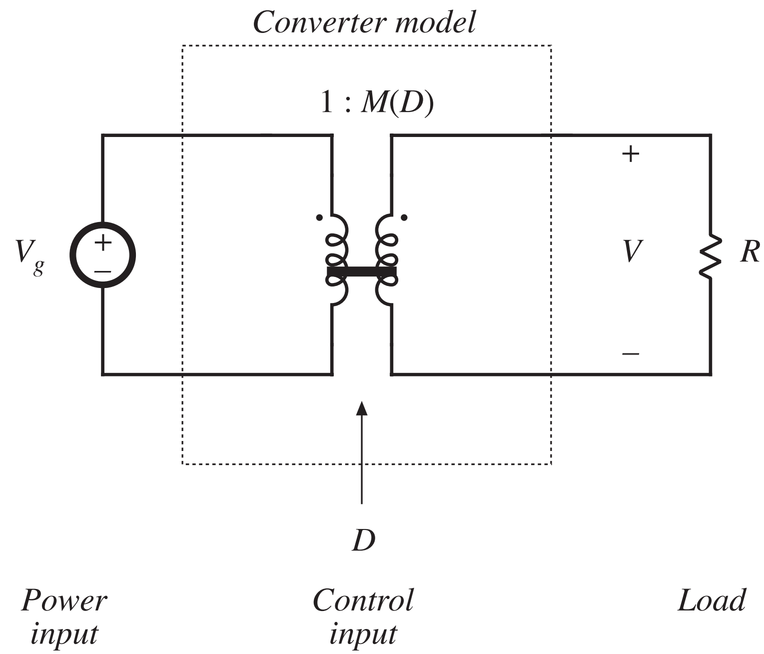
Hence, we expect their equivalent circuit models to be qualitatively similar.

Canonical model:

- A standard form of equivalent circuit model, which represents the above physical properties
- Plug in parameter values for a given specific converter

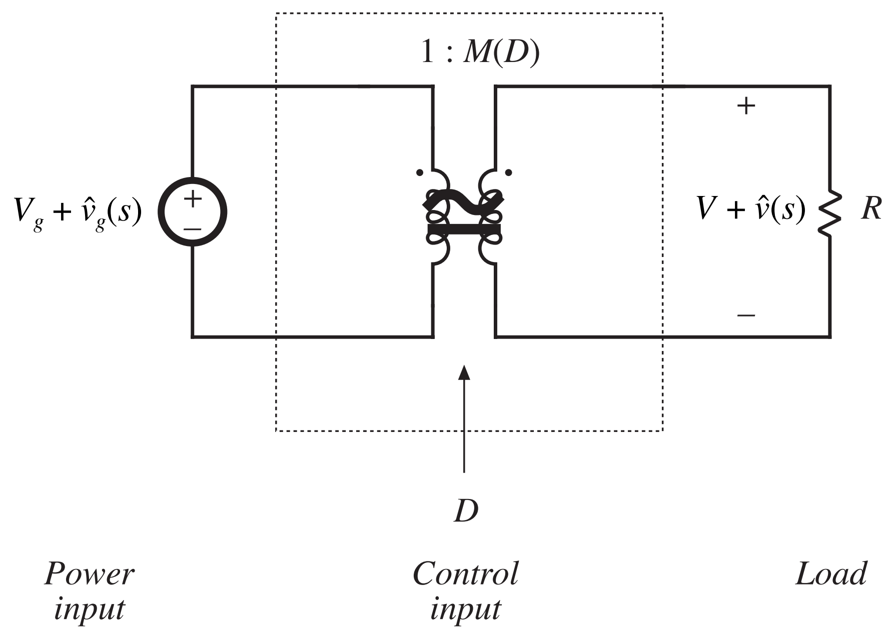
7.5.1. Development of the canonical circuit model

1. Transformation of dc voltage and current levels
- modeled as in Chapter 3 with ideal dc transformer
- effective turns ratio $M(D)$
- can refine dc model by addition of effective loss elements, as in Chapter 3



Steps in the development of the canonical circuit model

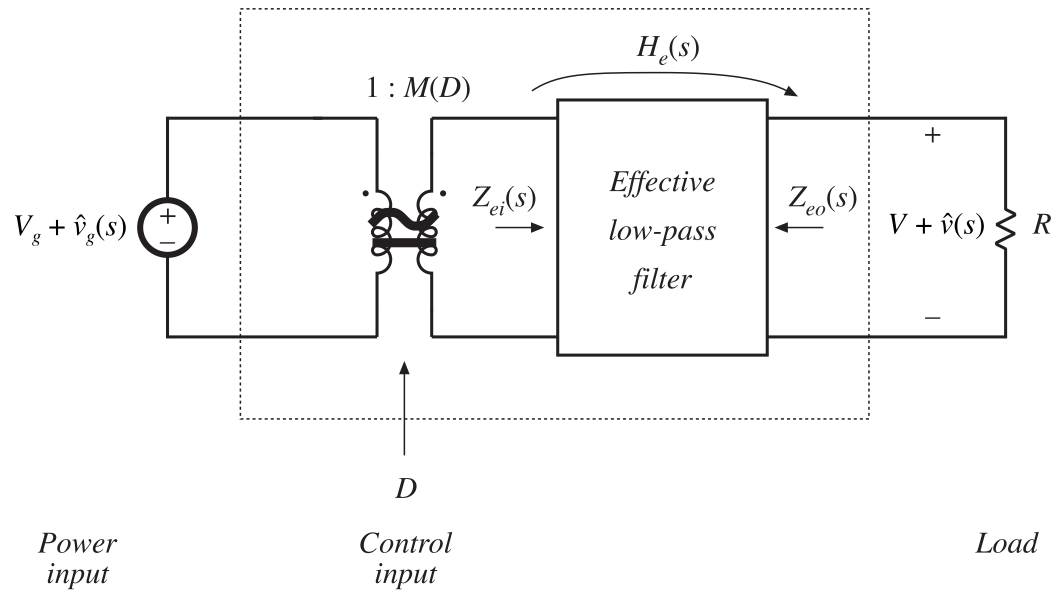
2. Ac variations in $v_g(t)$ induce ac variations in $v(t)$
- these variations are also transformed by the conversion ratio $M(D)$



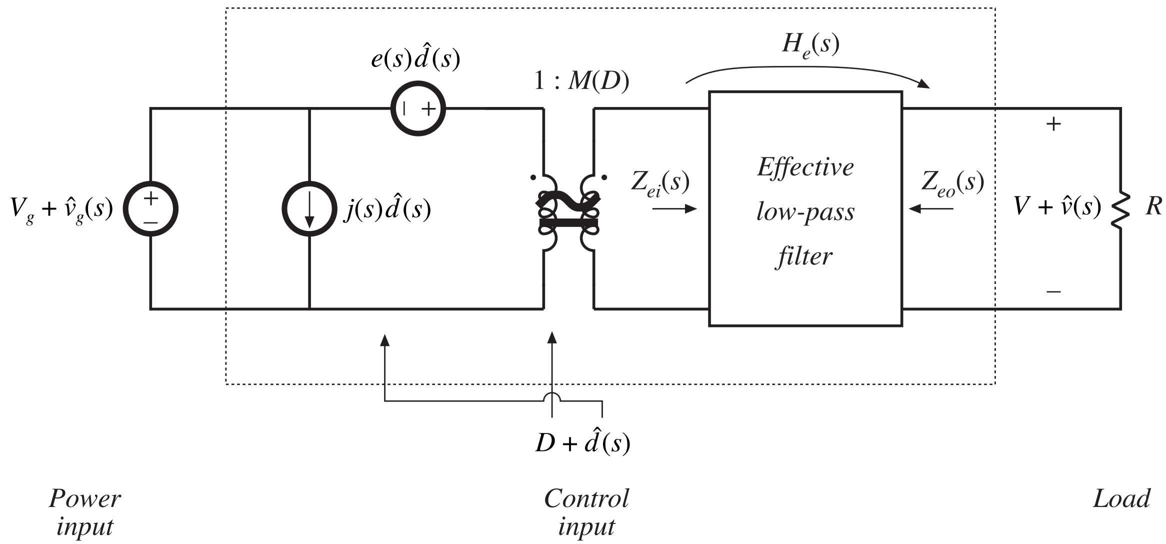
Steps in the development of the canonical circuit model

3. Converter must contain an effective low-pass filter characteristic

- necessary to filter switching ripple
- also filters ac variations
- effective filter elements may not coincide with actual element values, but can also depend on operating point



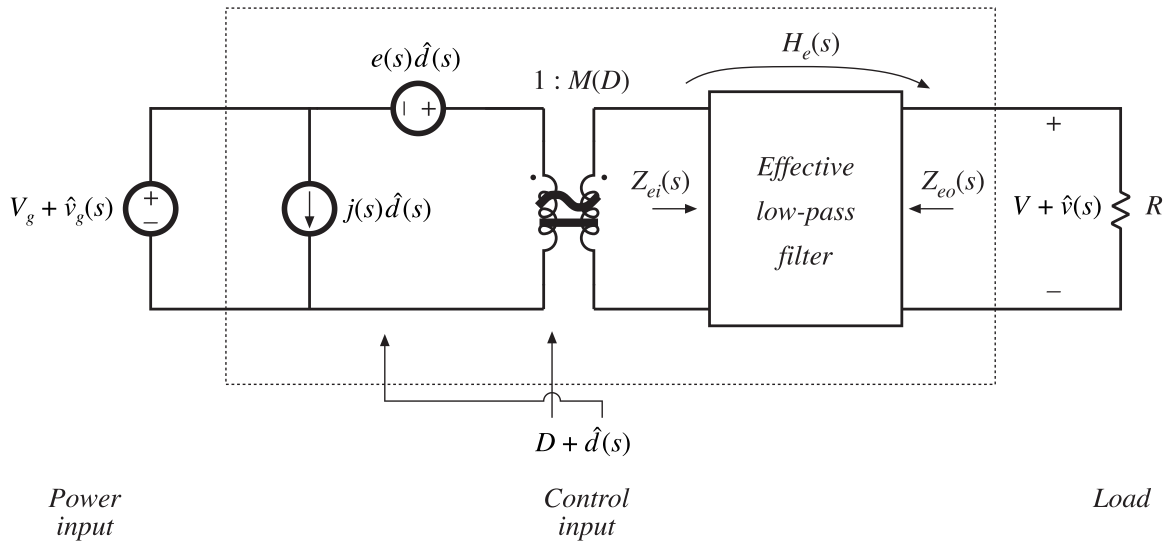
Steps in the development of the canonical circuit model



4. Control input variations also induce ac variations in converter waveforms

- Independent sources represent effects of variations in duty cycle
- Can push all sources to input side as shown. Sources may then become frequency-dependent

Transfer functions predicted by canonical model

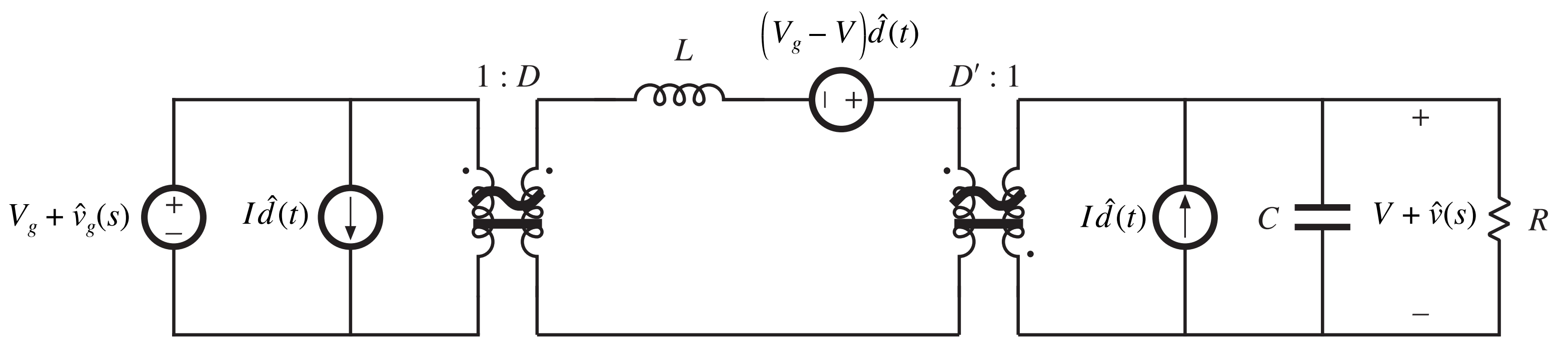


Line-to-output transfer function: $$G_{vg}(s) = \frac{\hat{v}(s)}{\hat{v}_g(s)} = M(D)\, H_e(s)$$

Control-to-output transfer function: $$G_{vd}(s) = \frac{\hat{v}(s)}{\hat{d}(s)} = e(s)\, M(D)\, H_e(s)$$

7.5.2 Example: manipulation of the buck-boost converter model into canonical form

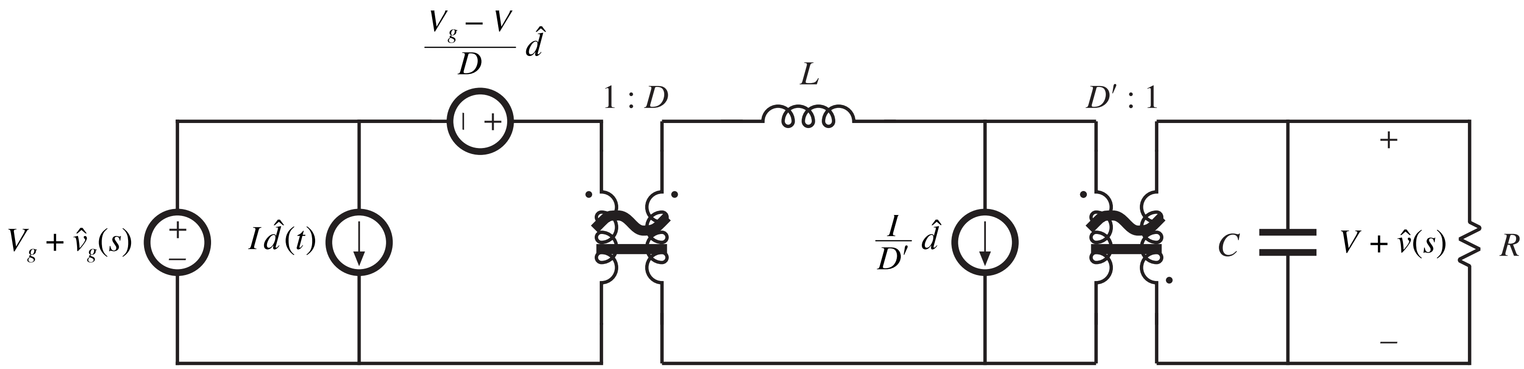
Small-signal ac model of the buck-boost converter



- Push independent sources to input side of transformers
- Push inductor to output side of transformers
- Combine transformers

Step 1

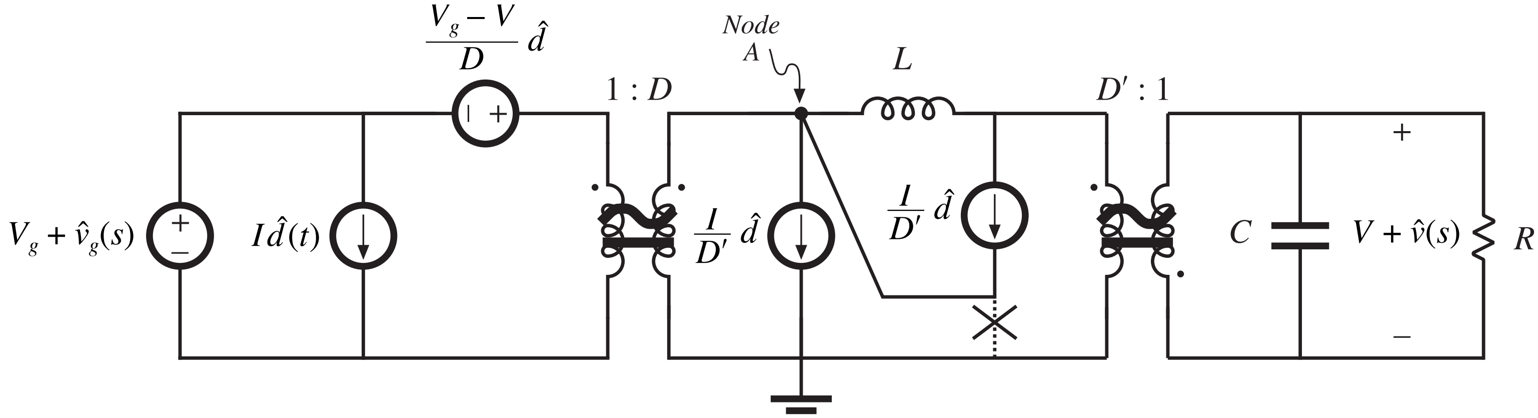
- Push voltage source through $1{:}D$ transformer
- Move current source through $D'{:}1$ transformer



Step 2

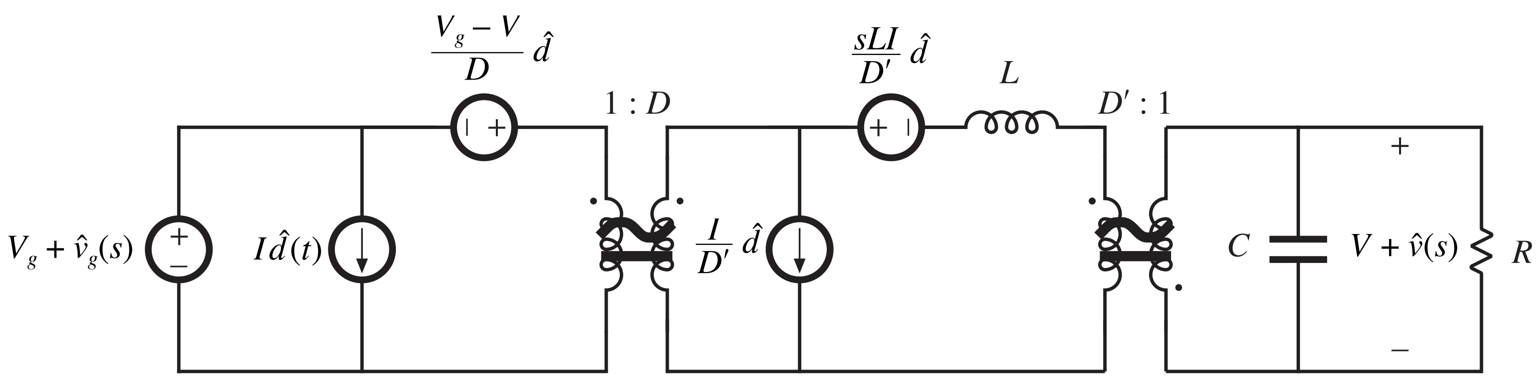
How to move the current source past the inductor:

- Break ground connection of current source, and connect to node *A* instead.
- Connect an identical current source from node *A* to ground, so that the node equations are unchanged.



Step 3

The parallel-connected current source and inductor can now be replaced by a Thevenin-equivalent network:

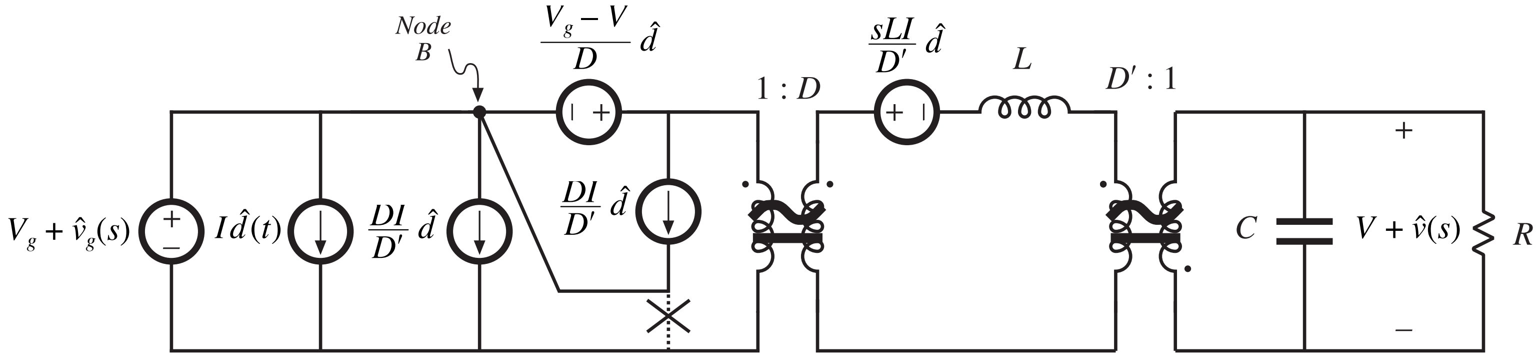


Step 4

Now push current source through $1{:}D$ transformer.

Push current source past voltage source, again by:

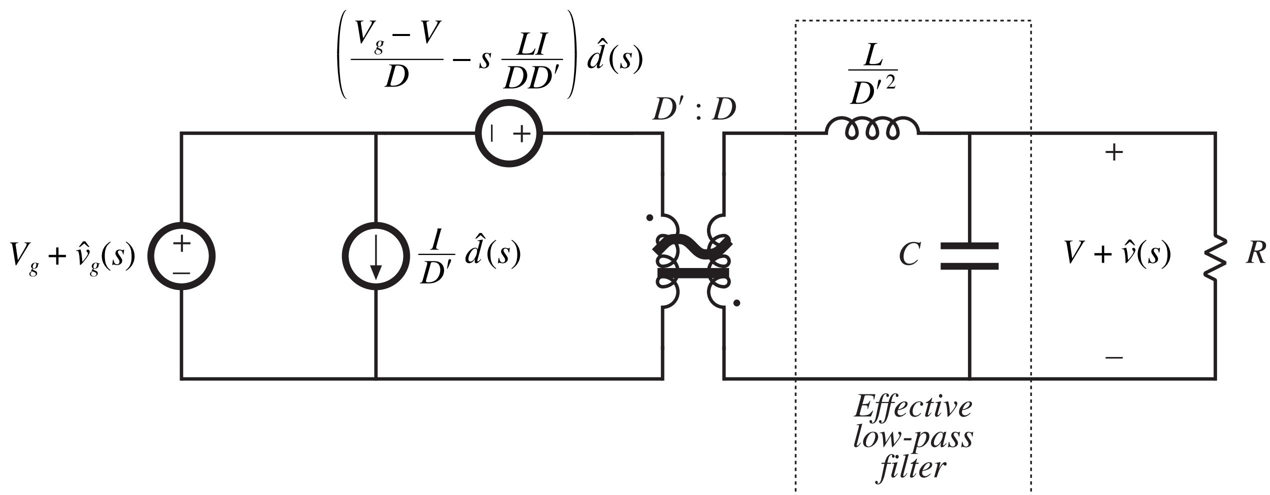
- Breaking ground connection of current source, and connecting to node *B* instead.
- Connecting an identical current source from node *B* to ground, so that the node equations are unchanged.
- Note that the resulting parallel-connected voltage and current sources are equivalent to a single voltage source.



Step 5: final result

Push voltage source through 1:D transformer, and combine with existing input-side transformer.

Combine series-connected transformers.



Coefficient of control-input voltage generator

Voltage source coefficient is:

$$e(s) = \frac{V_g + V}{D} - \frac{s\,LI}{D\,D'}$$

Simplification, using dc relations, leads to

$$e(s) = -\frac{V}{D^2}\left(1 - \frac{s\,DL}{D'^2\,R}\right)$$

Pushing the sources past the inductor causes the generator to become frequency-dependent.

7.5.3 Canonical circuit parameters for some common converters

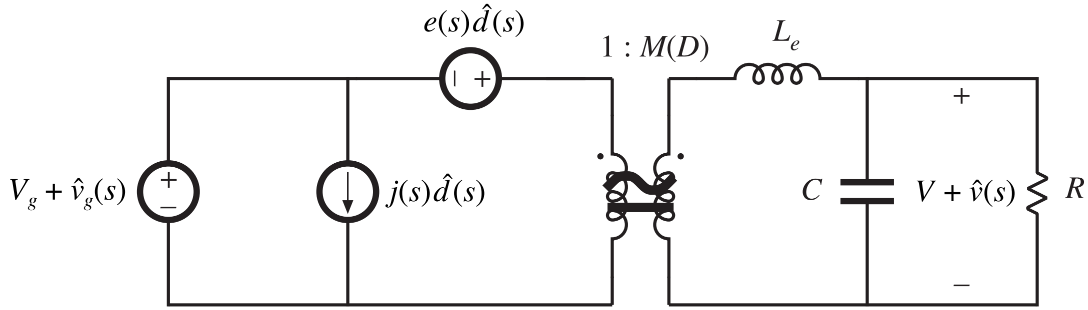


Table 7.1. Canonical model parameters for the ideal buck, boost, and buck-boost converters

Converter	*M(D)*	L_e	*e(s)*	*j(s)*
Buck	D	L	$\frac{V}{D^2}$	$\frac{V}{R}$
Boost	$\frac{1}{D'}$	$\frac{L}{D'^2}$	$V\left(1 - \frac{s\,L}{D'^2\,R}\right)$	$\frac{V}{D'^2\,R}$
Buck-boost	$-\frac{D}{D'}$	$\frac{L}{D'^2}$	$-\frac{V}{D^2}\left(1 - \frac{s\,DL}{D'^2\,R}\right)$	$-\frac{V}{D'^2\,R}$