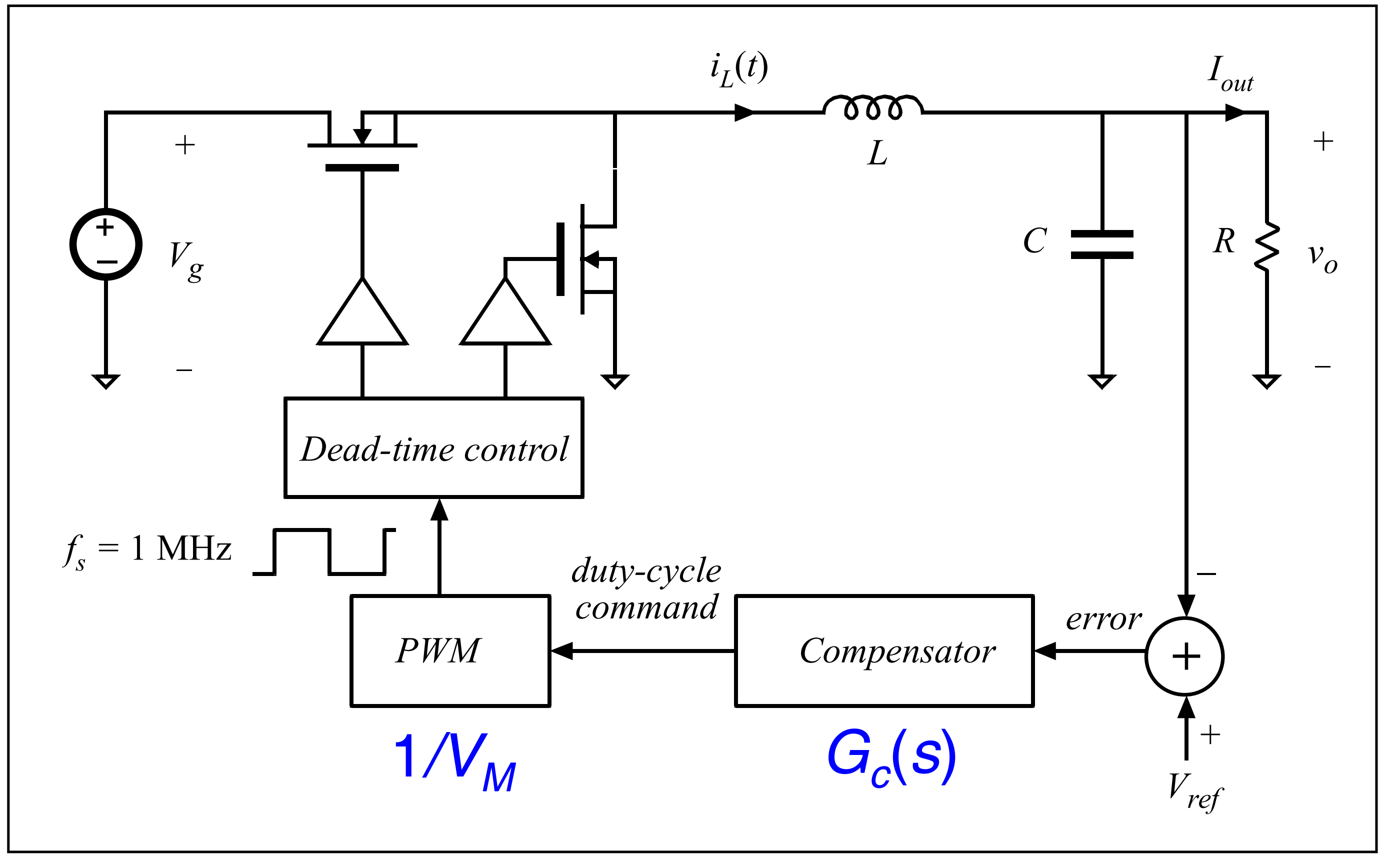
Another Compensator Design Example

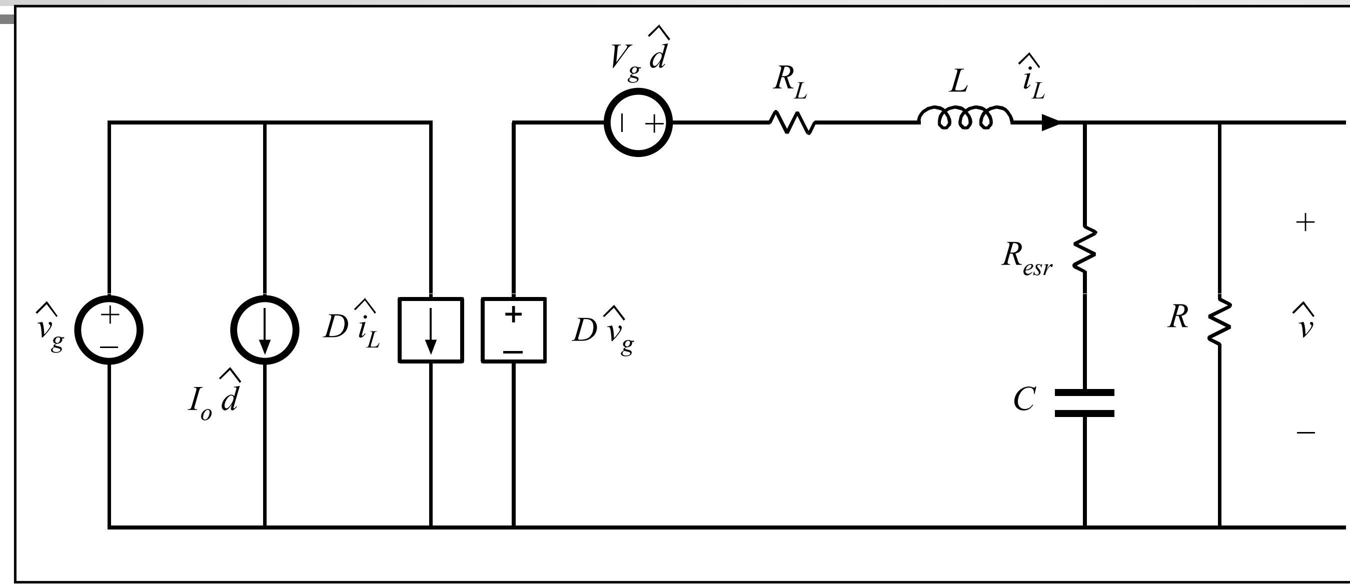


Point-of-Load Synchronous Buck Regulator

Power stage parameters

- Switching frequency: f_s = 1MHz
- V_{ref} = 1.8 V
- I_{out} = 0 to 5 A
- V_g = 5 V
- L = 1 μH
- R_L = 30 mΩ
- C = 200 μF
- R_{esr} = 0.8 mΩ
- V_M = 1 V
- H = 1

Buck Averaged Small-Signal Model



$$G_{vd}(s) = \frac{\hat{v}_o}{\hat{d}}$$

$$G_{vd}(s) = V_g \frac{1 + \frac{s}{\omega_{esr}}}{1 + \frac{1}{Q}\frac{s}{\omega_o} + \left(\frac{s}{\omega_o}\right)^2}$$

Pair of poles:

$$f_o = \frac{1}{2\pi\sqrt{CL}} = 11\,\text{kHz}$$

$$Q_{loss} = \frac{\sqrt{L/C}}{R_{esr} + R_L} = 2.3 \to 7.2\text{ dB} \qquad Q_{load} = \frac{R}{\sqrt{L/C}} > 5$$

$$Q = Q_{loss} \parallel Q_{load} = \frac{Q_{loss}Q_{load}}{Q_{loss} + Q_{load}} < 2.3 \to 7.2\text{ dB}$$

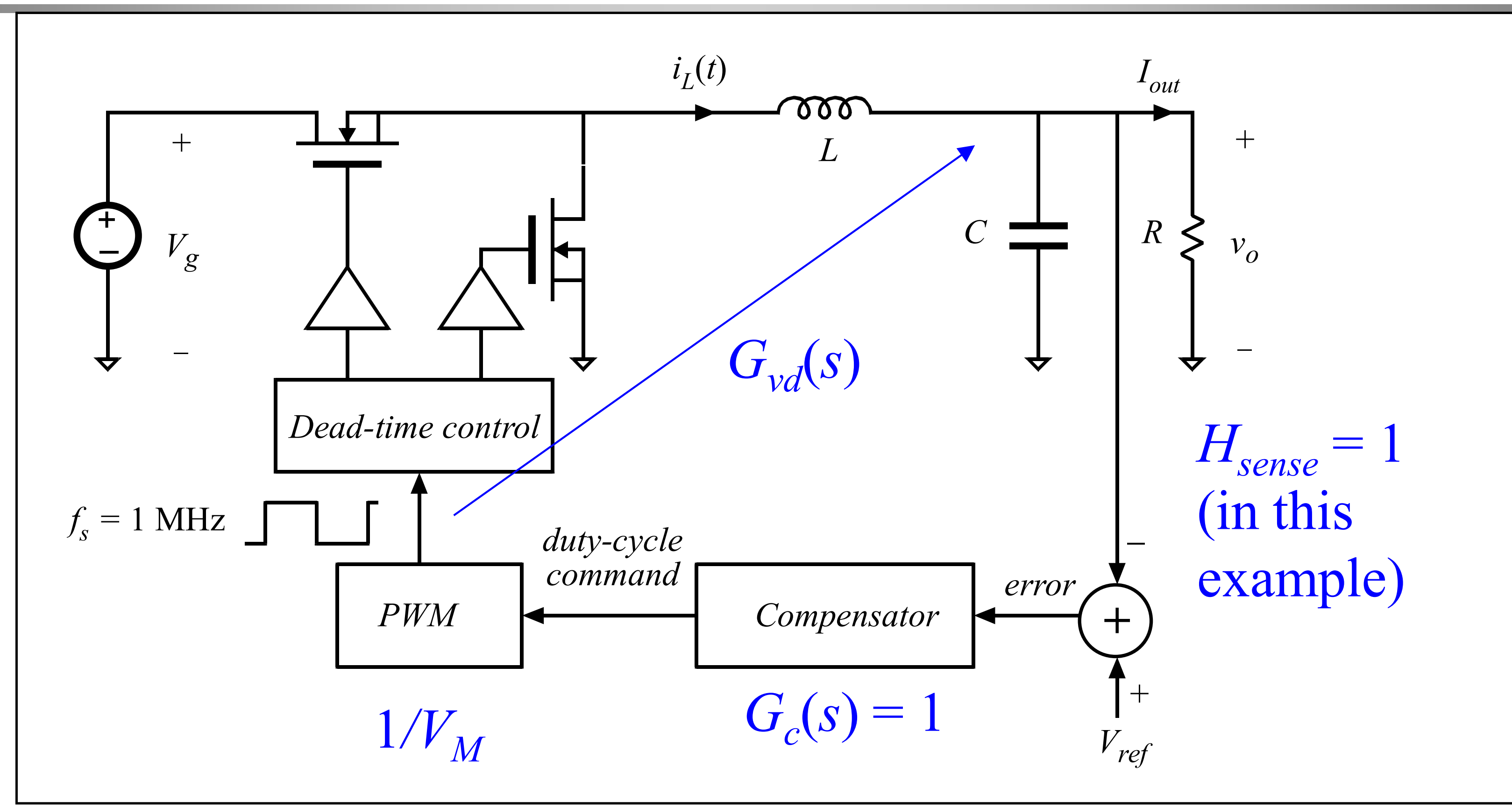
Low-frequency gain (including PWM gain):

$$G_{vdo}\frac{1}{V_M} = 5 \to 14\text{dB}$$

ESR zero:

$$f_{esr} = \frac{1}{2\pi C R_{esr}} = 1\,\text{MHz}$$

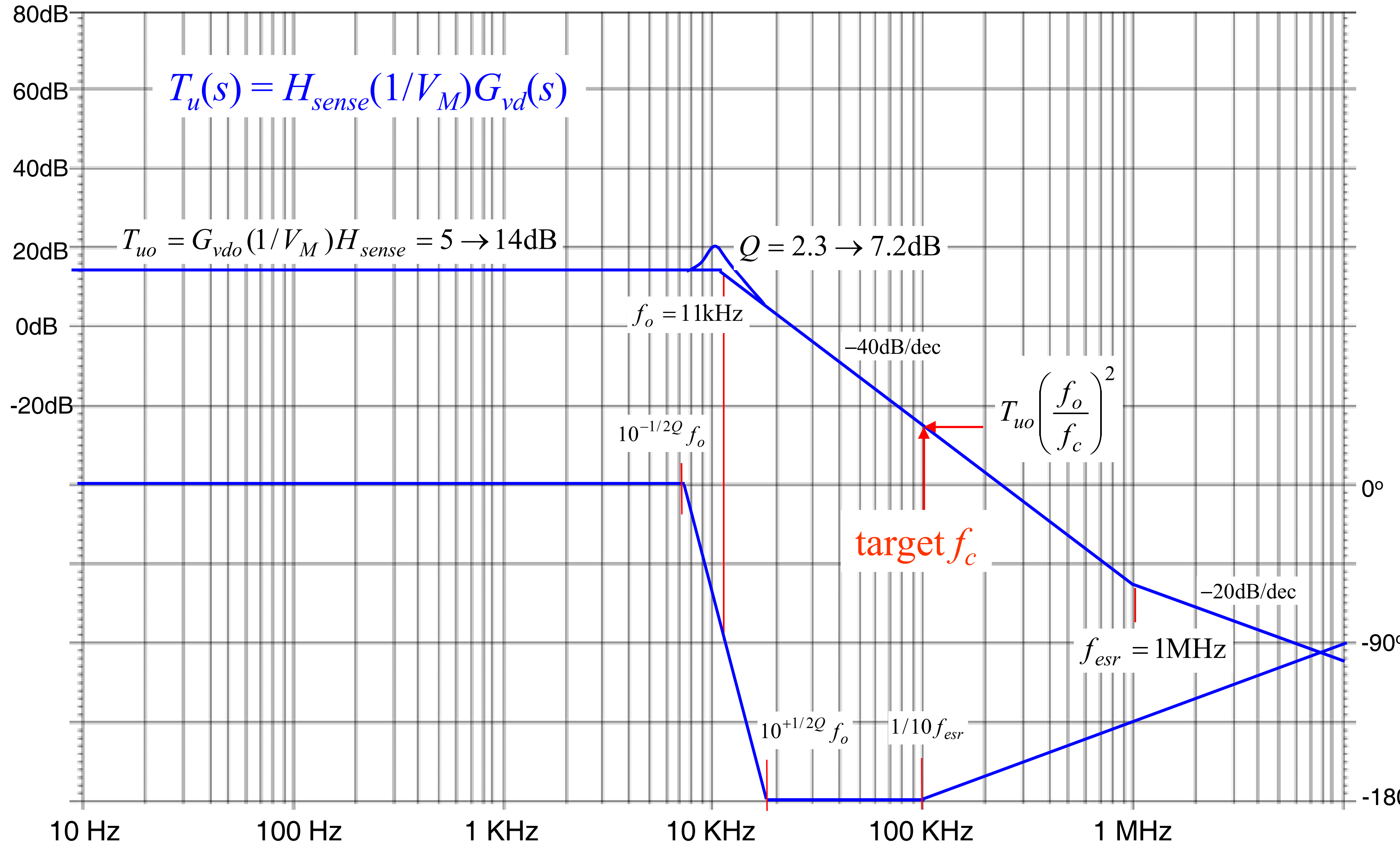
Uncompensated loop gain T_u



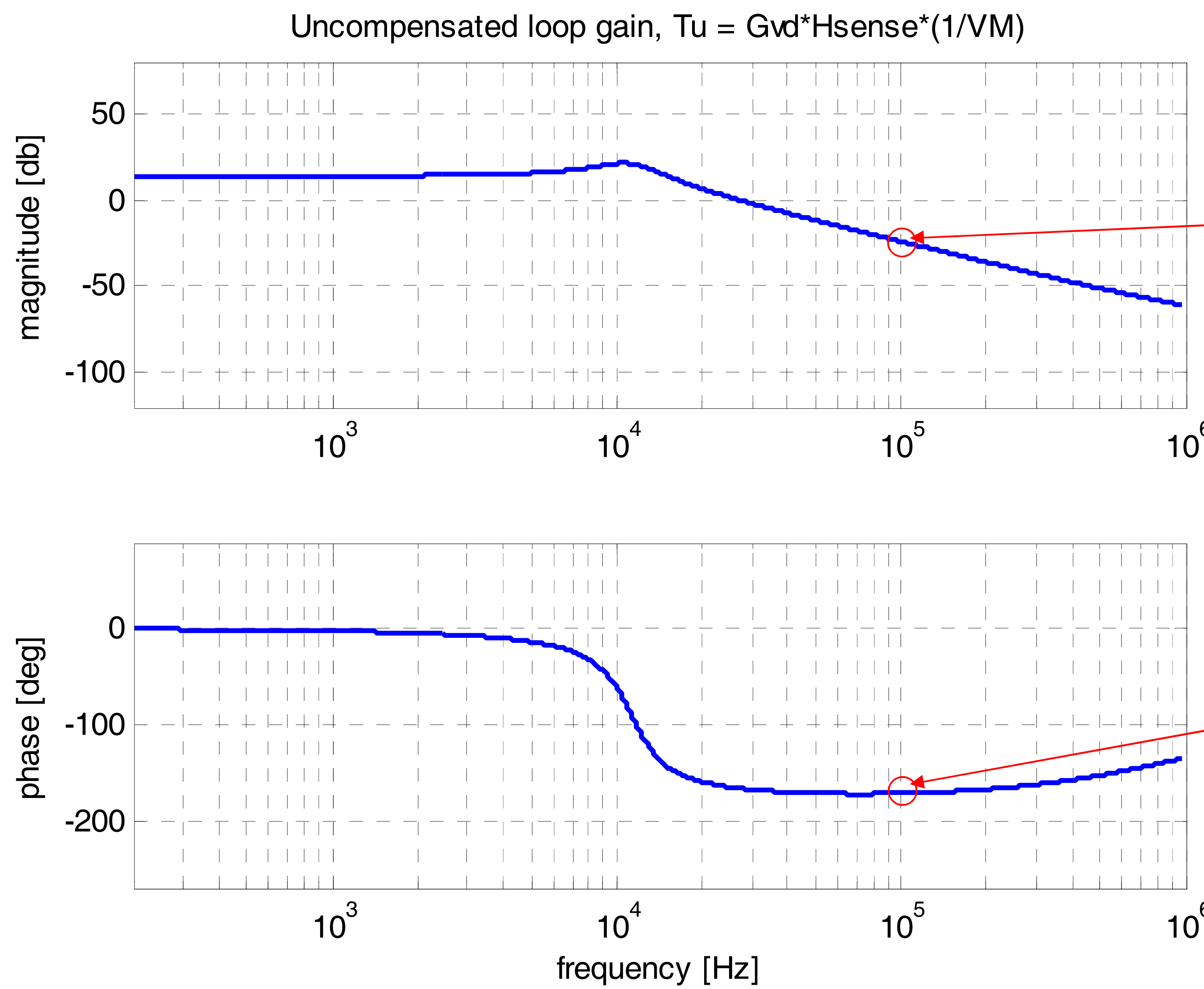
$$T_u(s) = H_{sense}(1/V_M)G_{vd}(s)$$

Plot magnitude and phase responses of $T_u(s)$ to plan how to design $G_c(s)$

Magnitude and phase Bode plots of T_u



Magnitude and phase Bode plots of T_u



Exact magnitude and phase responses (MATLAB)

Target cross-over frequency $f_c = f_s/10 = 100$ kHz

No phase margin: a lead (PD) compensator is required

Lead (PD) compensator design

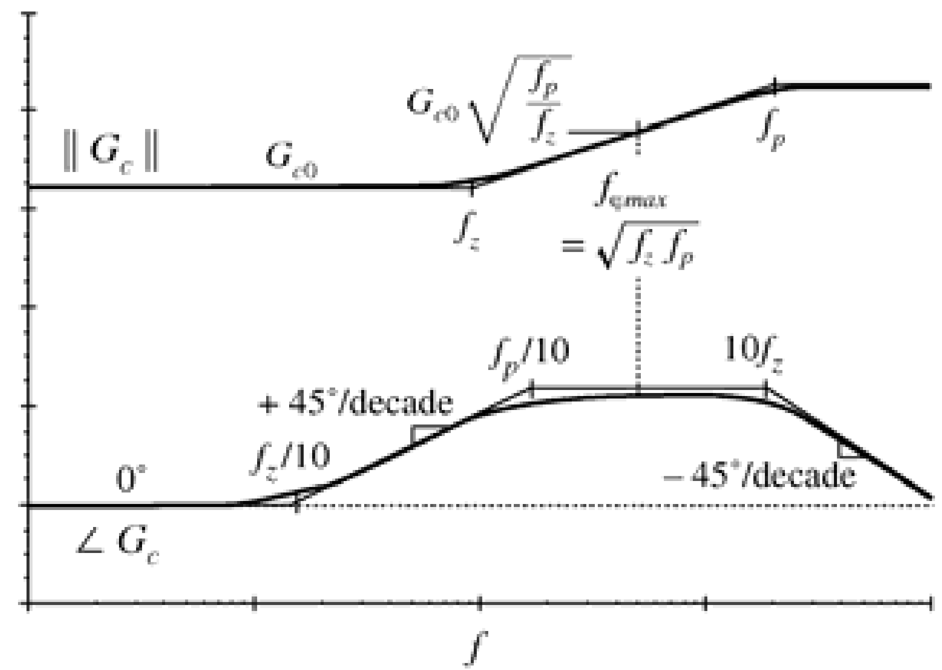
1. Choose: $f_c = 100\ \text{kHz}$

$\theta = \varphi_m = 53^o$

2. Compute:

$$f_z = f_c \sqrt{\frac{1-\sin(\theta)}{1+\sin(\theta)}} = 33\ \text{kHz}$$

$$f_p = f_c \sqrt{\frac{1+\sin(\theta)}{1-\sin(\theta)}} = 300\ \text{kHz}$$



3. Find G_{co} to position the crossover frequency:

$$\underbrace{T_{uo}\left(\frac{f_o}{f_c}\right)^2}_{\text{Magnitude of } T_u \text{ at } f_c} \underbrace{G_{co}\sqrt{\frac{f_p}{f_z}}}_{\text{Magnitude of } G_c \text{ at } f_c} = 1 \;\longrightarrow\; G_{co} = \frac{1}{T_{uo}}\left(\frac{f_c}{f_o}\right)^2 \sqrt{\frac{f_z}{f_p}} = 5.45 \rightarrow 15\ \text{dB}$$

Lead (PD) compensator summary

$$G_c(s) = G_{co} \underbrace{\frac{\left(1+\frac{s}{\omega_z}\right)}{\left(1+\frac{s}{\omega_{p1}}\right)}}_{\text{Lead compensator}} \underbrace{\frac{1}{\left(1+\frac{s}{\omega_{p2}}\right)}}_{\text{HF pole}}$$

$$G_{co} = 5.45 \rightarrow 15\ \mathrm{dB}$$

$$f_z = 33\ \mathrm{kHz}$$

$$f_{p1} = 300\ \mathrm{kHz}$$

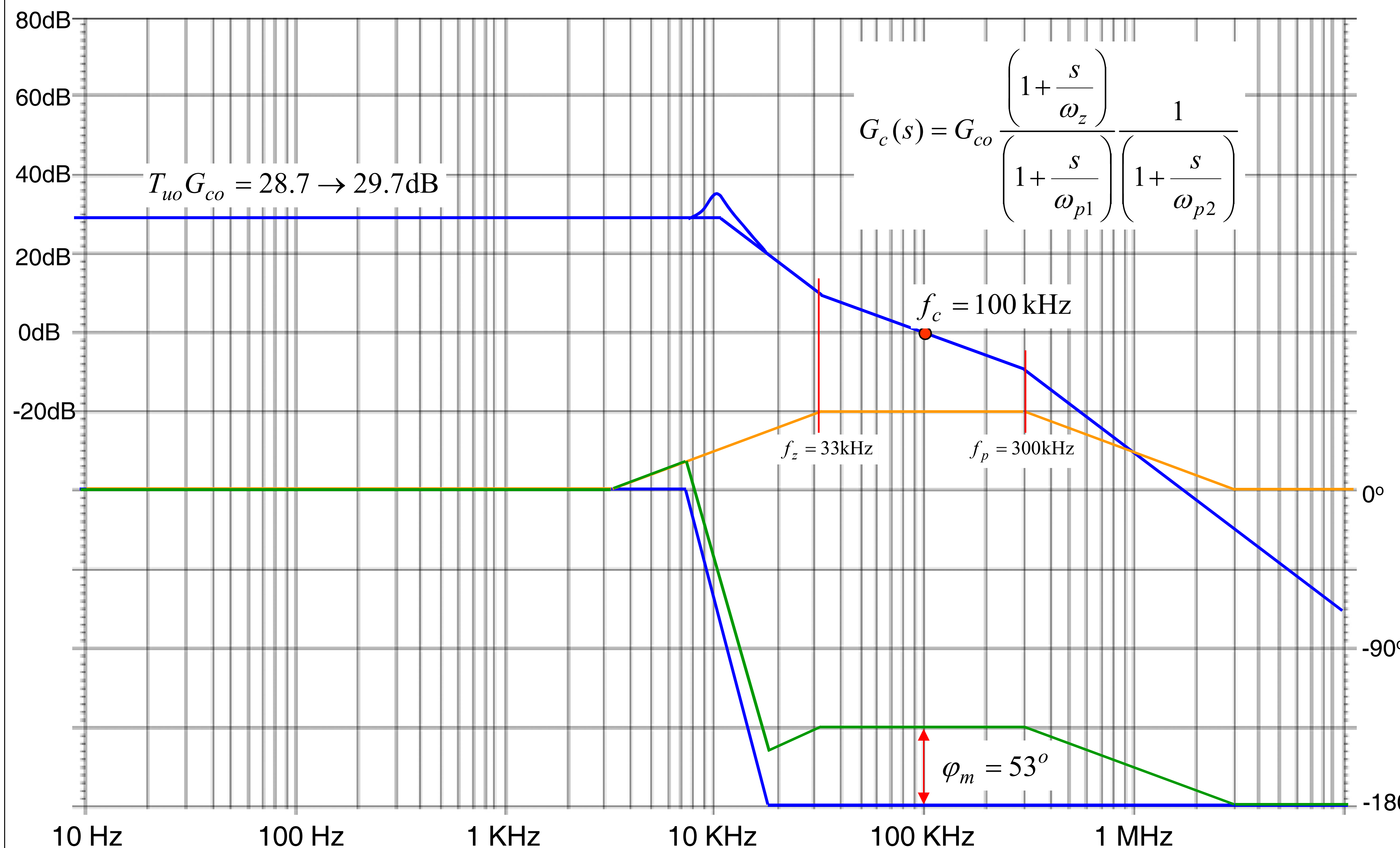
$f_c = 100\ \mathrm{kHz}$ (=1/10 of f_s)

High-frequency gain of the lead compensator: $G_{co} f_{p1}/f_z = 49$ (34 dB)

Added high-frequency pole: $f_{p2} = 1\ \mathrm{MHz}$ $(= f_{esr} = f_s$ in this example)

Practical implementation would require an op-amp with a gain bandwidth product of at least 49*f_{p2} = 49 MHz

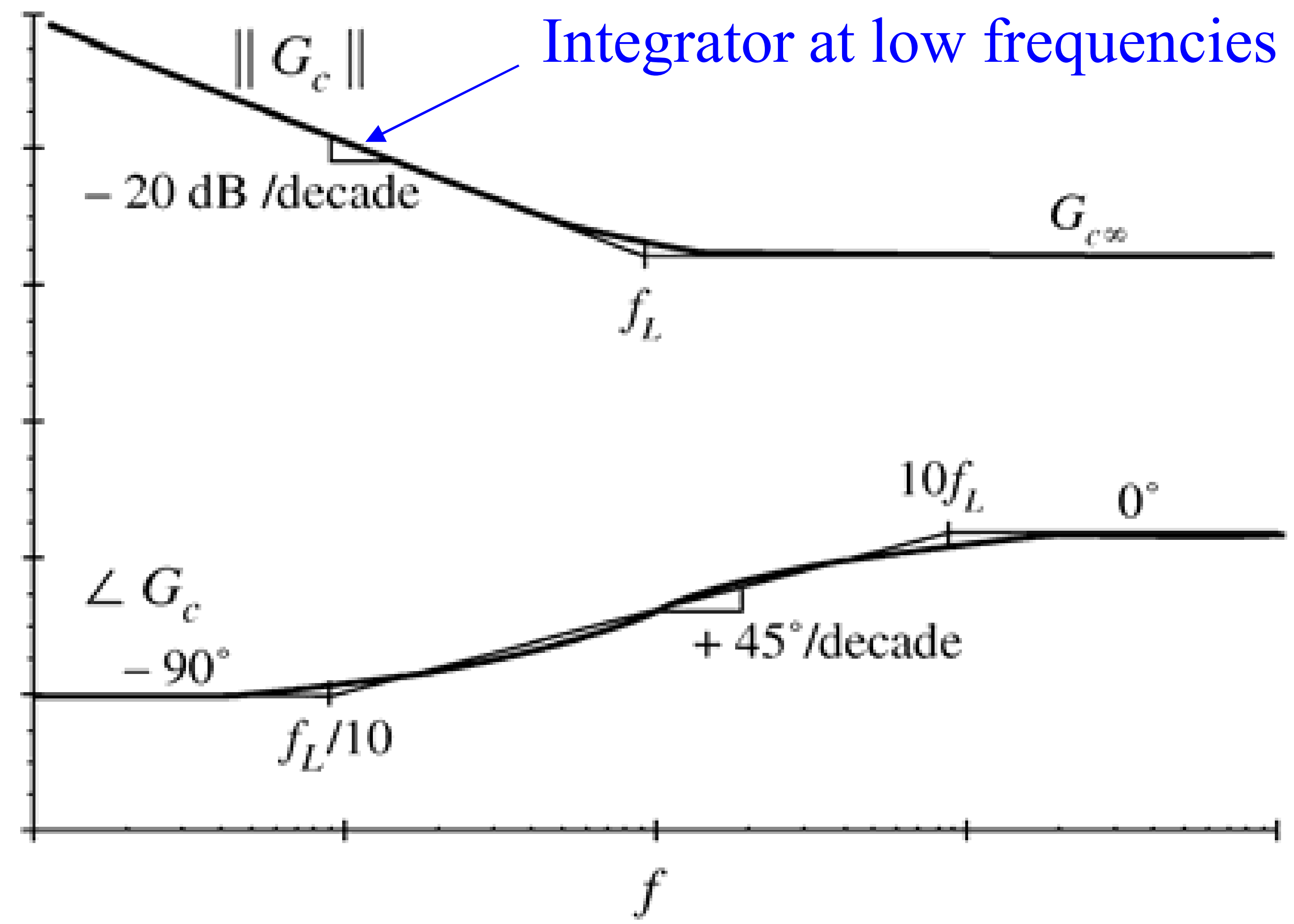
Loop gain with lead (PD) compensator



Add lag (PI) compensator

$$G_c(s) = G_{c\infty}\left(1 + \frac{\omega_L}{s}\right)$$

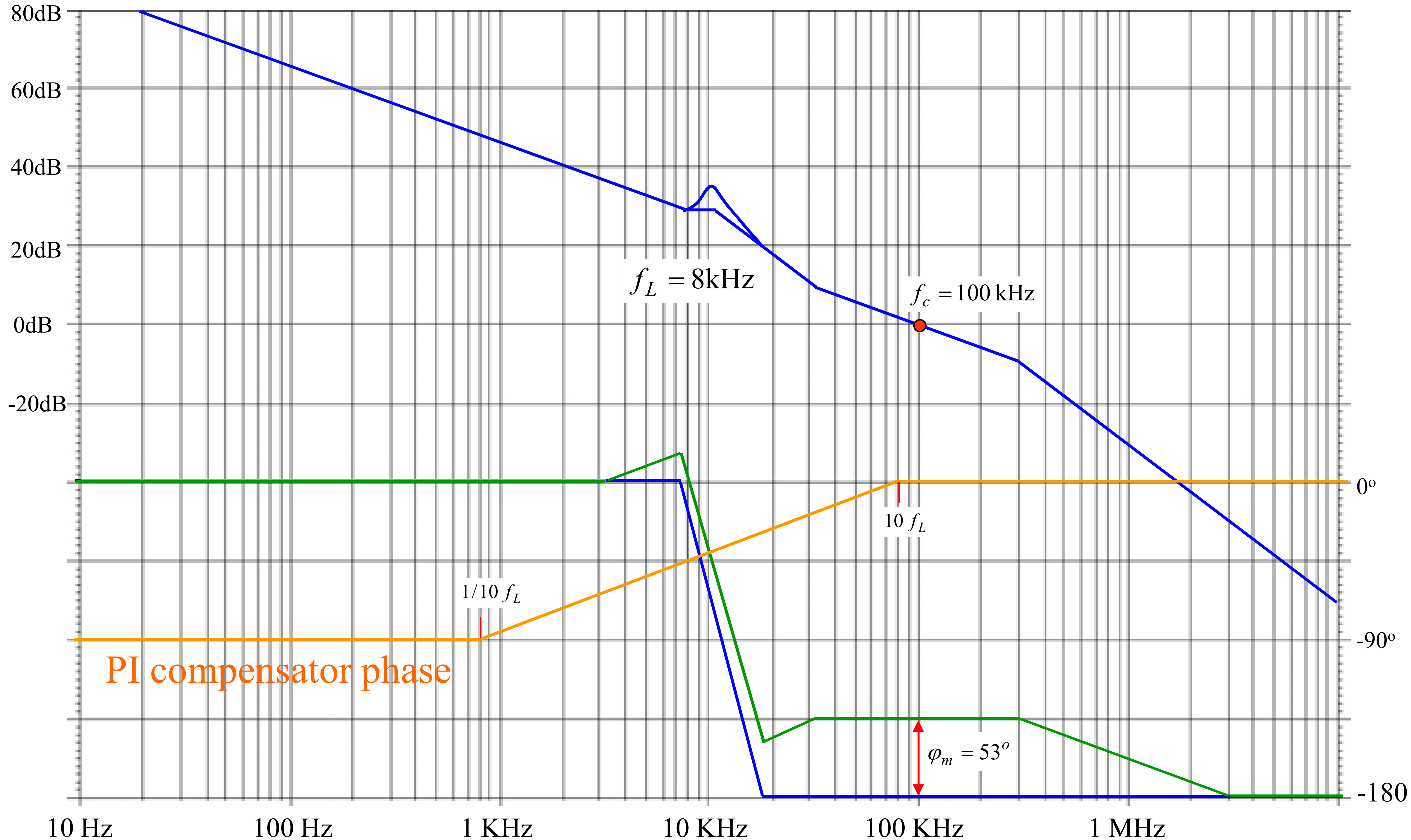
Improves low-frequency loop gain and regulation



Choose $10f_L < f_c$ so that phase margin stays approximately the same: $f_L = 8$ kHz

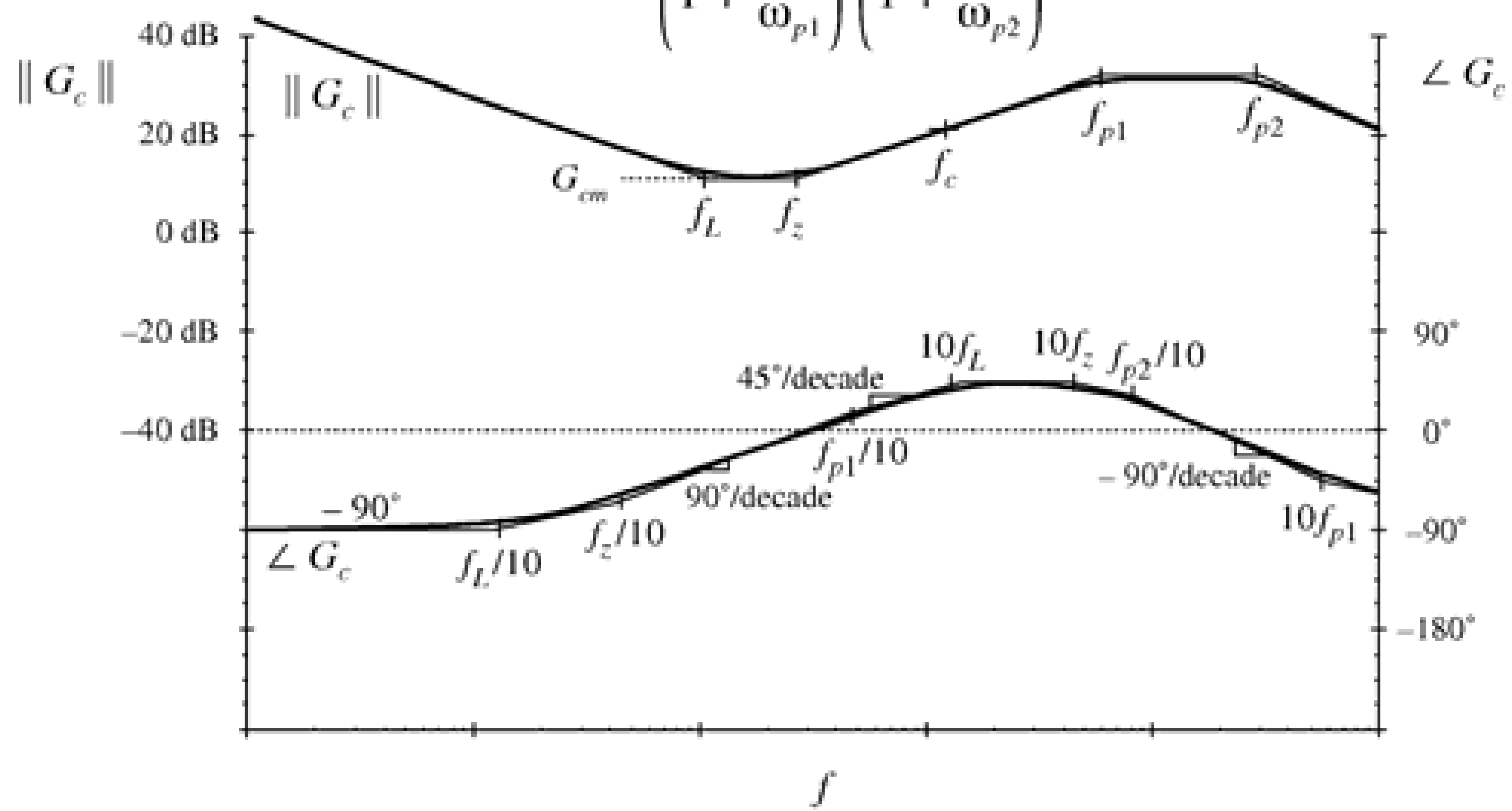
Keep the same cross-over frequency: $G_{c\infty} = G_{co} = G_{cm} = 5.45 \rightarrow 15$ dB

Adding PI Compensator



Complete analog PID compensator: summary

$$G_c(s) = G_{cm}\frac{\left(1+\frac{\omega_L}{s}\right)\left(1+\frac{s}{\omega_z}\right)}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)}$$



$$G_{cm} = 5.45 \rightarrow 15\ \text{dB}$$

$$f_L = 8\ \text{kHz}$$

$$f_z = 33\ \text{kHz}$$

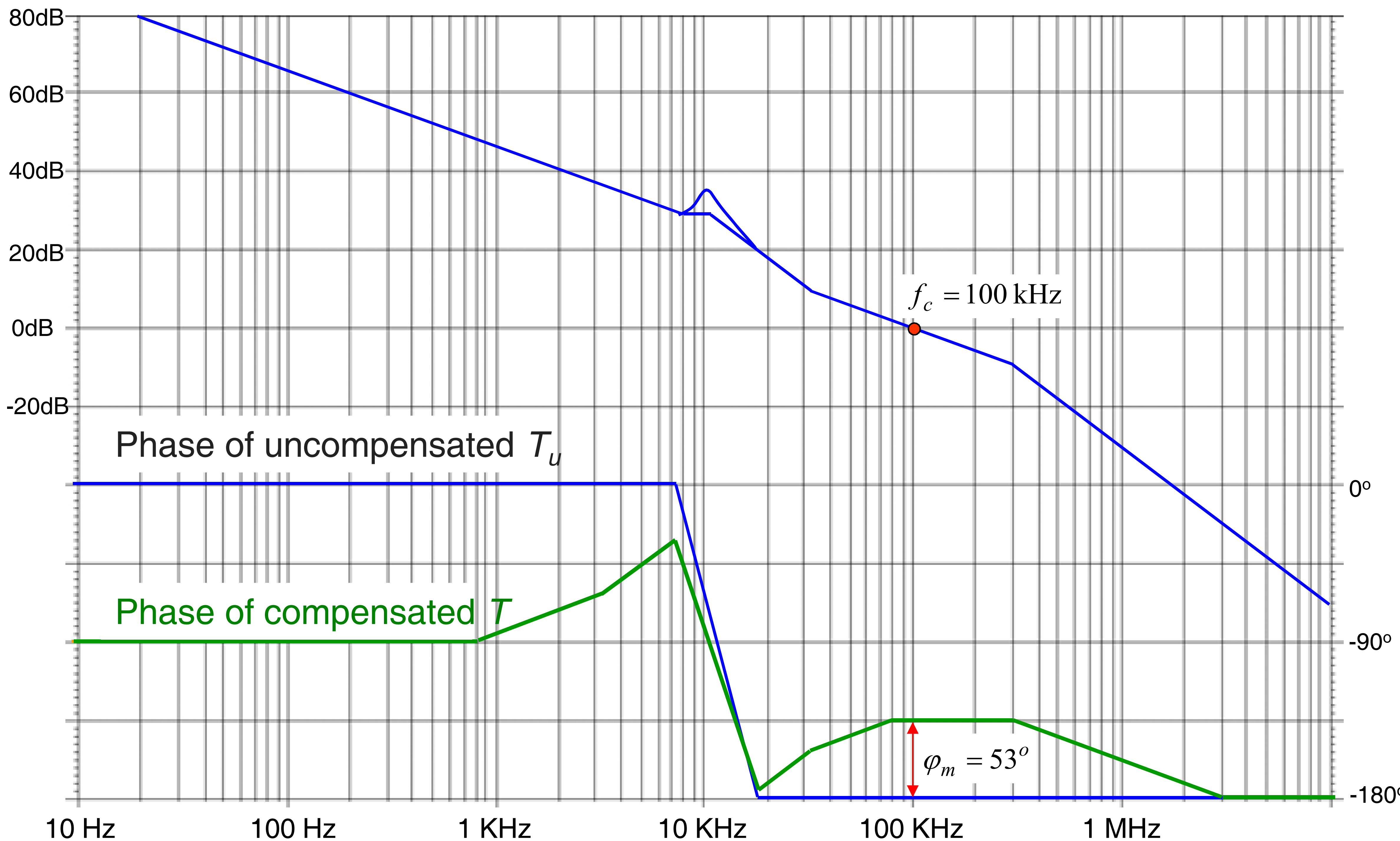
$$f_{p1} = 300\ \text{kHz}$$

$$f_{p2} = 1\ \text{MHz}$$

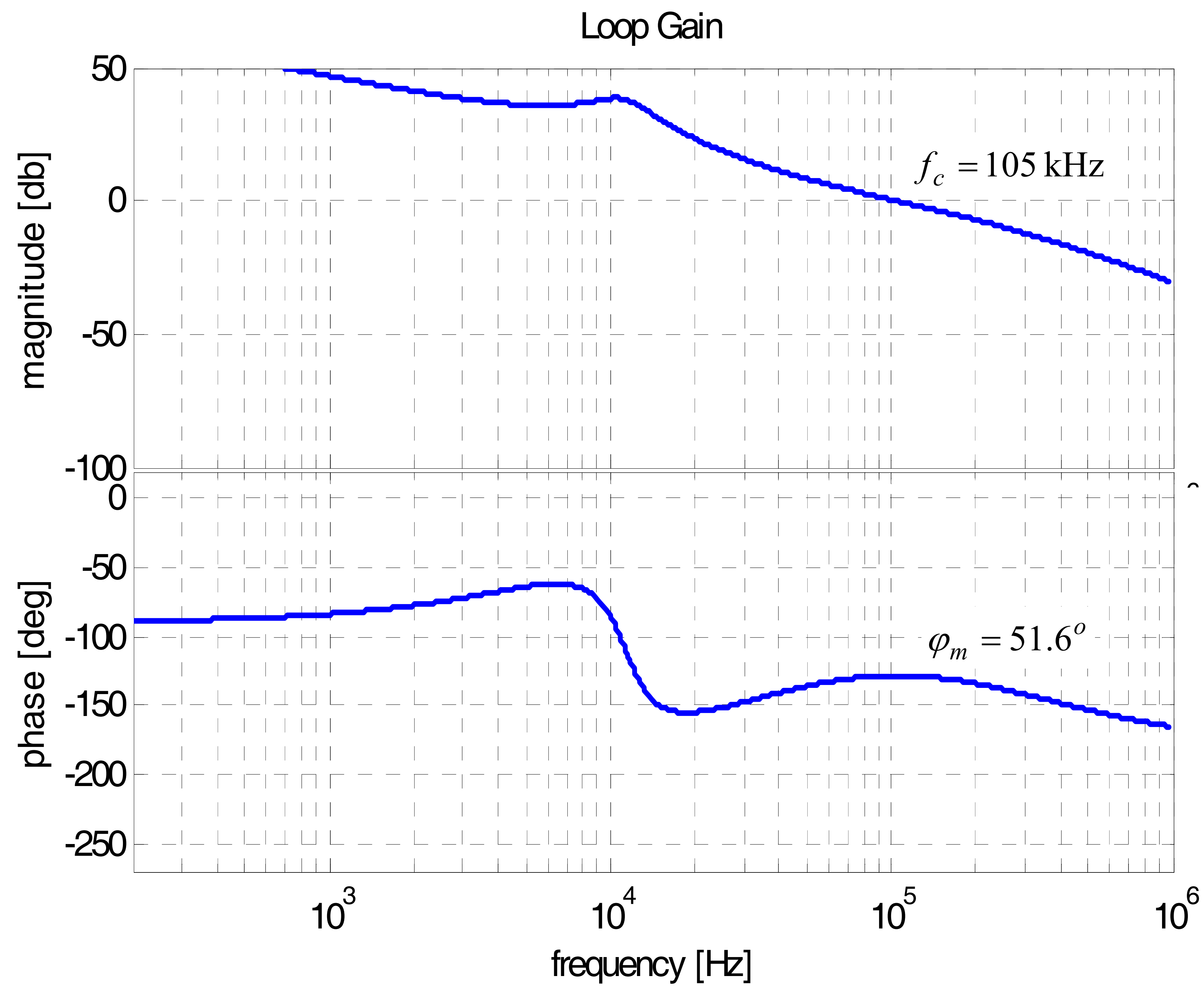
Crossover frequency: $f_c = 100\ \text{kHz}$ (=1/10 of f_s)

Phase margin: $\varphi_m = 53^o$

Magnitude and phase Bode plots of T



Verification: exact loop gain magnitude and phase responses (MATLAB)



Verification of closed-loop responses

Closed-loop reference-to-output response

Closed-loop transfer function from $\hat{v}_{ref}$ to $\hat{v}(s)$ is:

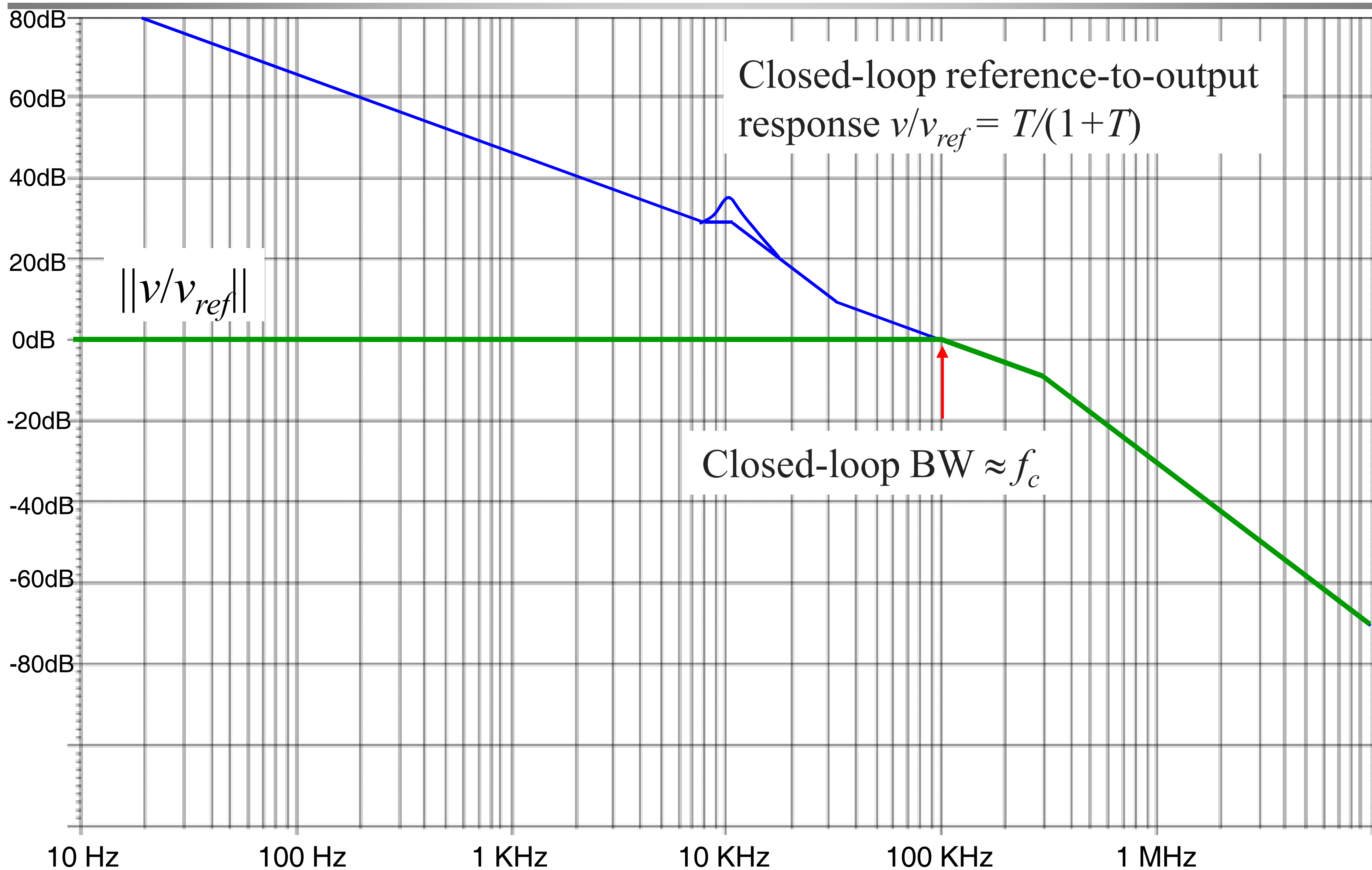
$$\left.\frac{\hat{v}(s)}{\hat{v}_{ref}(s)}\right|_{\substack{\hat{v}_g = 0 \\ \hat{i}_{load} = 0}} = \frac{1}{H(s)} \frac{T(s)}{1 + T(s)}$$

Closed-loop output impedance

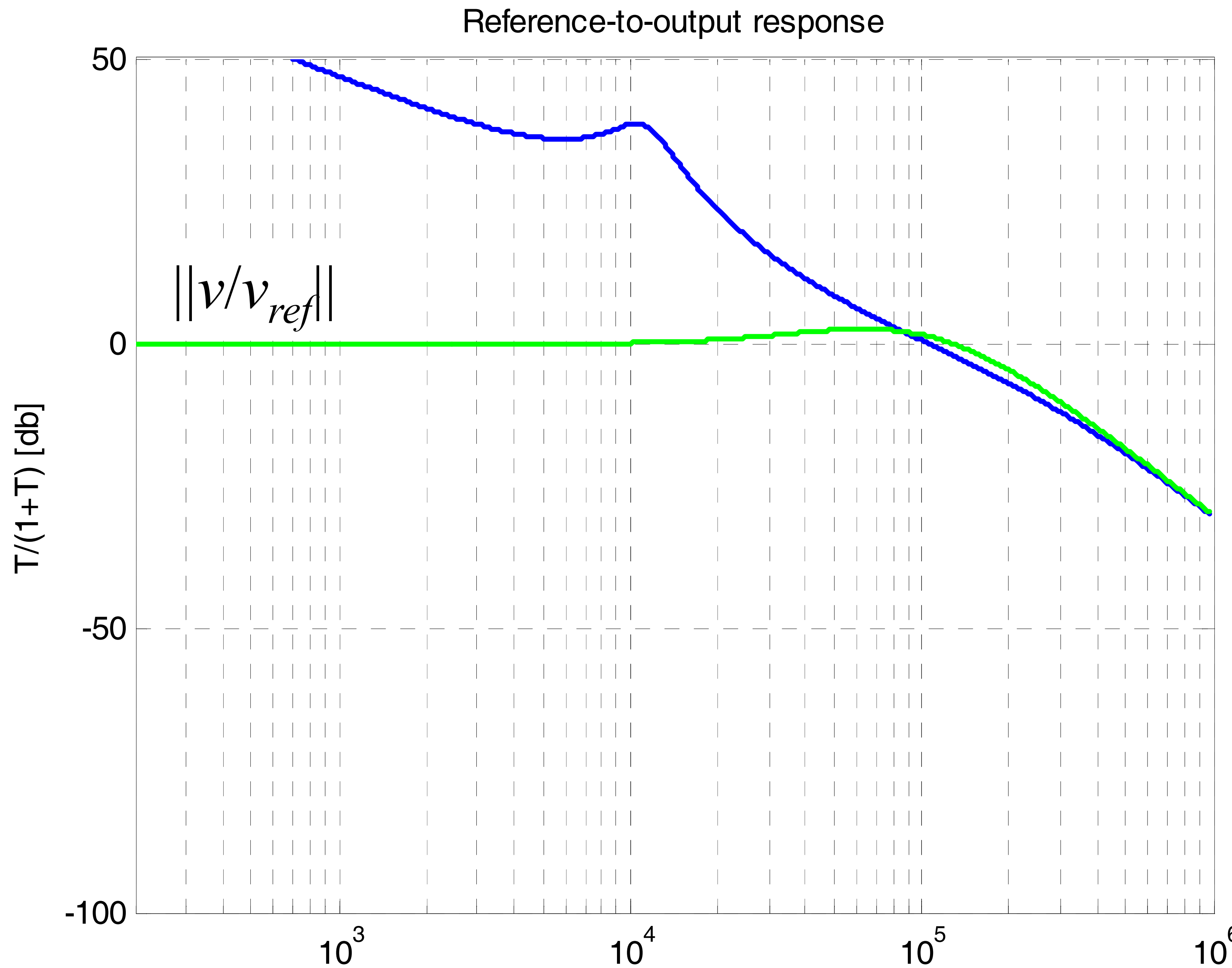
$$\left.\frac{\hat{v}(s)}{-\hat{i}_{load}(s)}\right|_{\substack{\hat{v}_{ref} = 0 \\ \hat{v}_g = 0}} = \frac{Z_{out}(s)}{1 + T(s)}$$

and step-load transient response

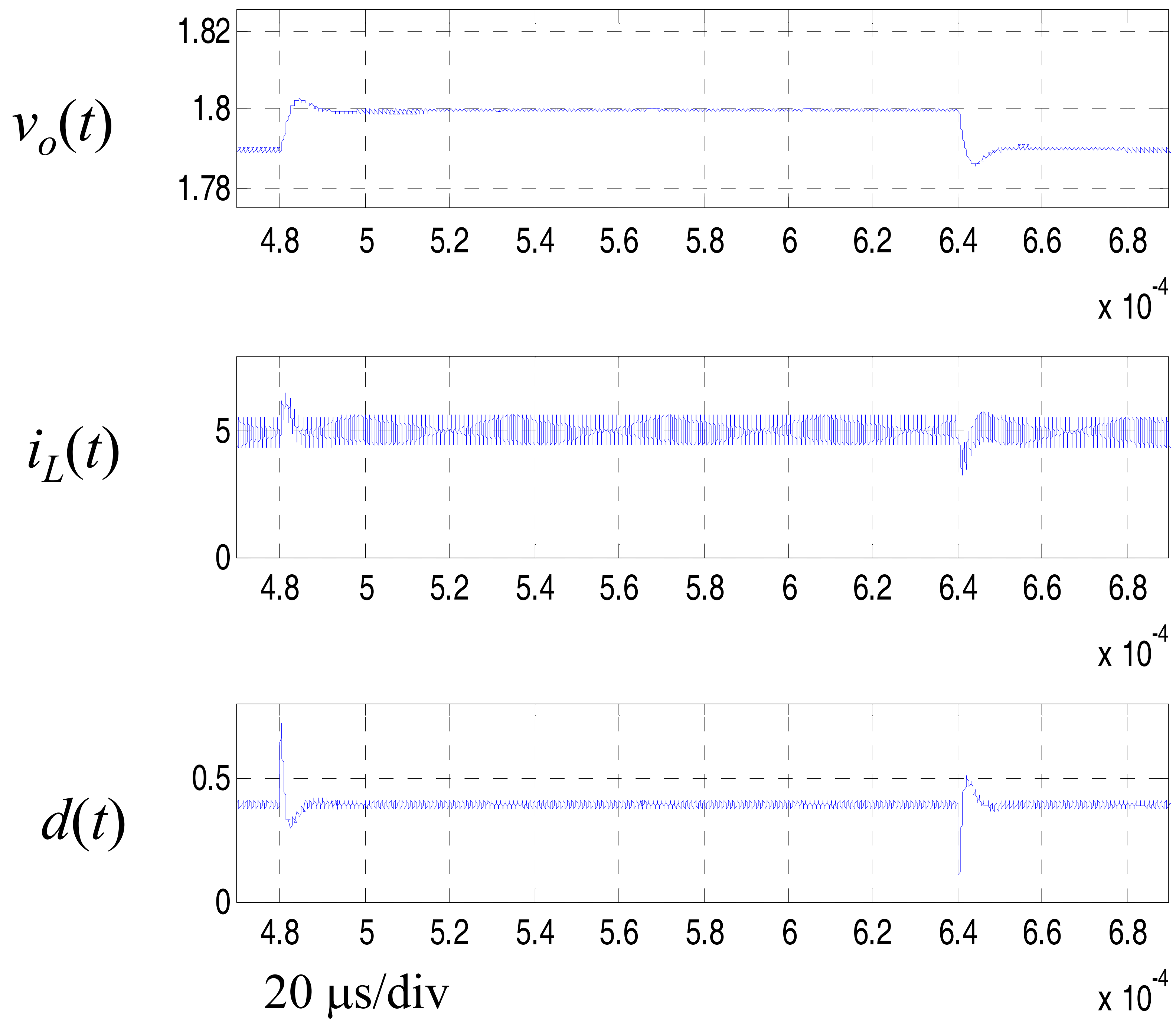
Construction of closed-loop $T/(1+T)$ response



Closed-loop reference-to-output response

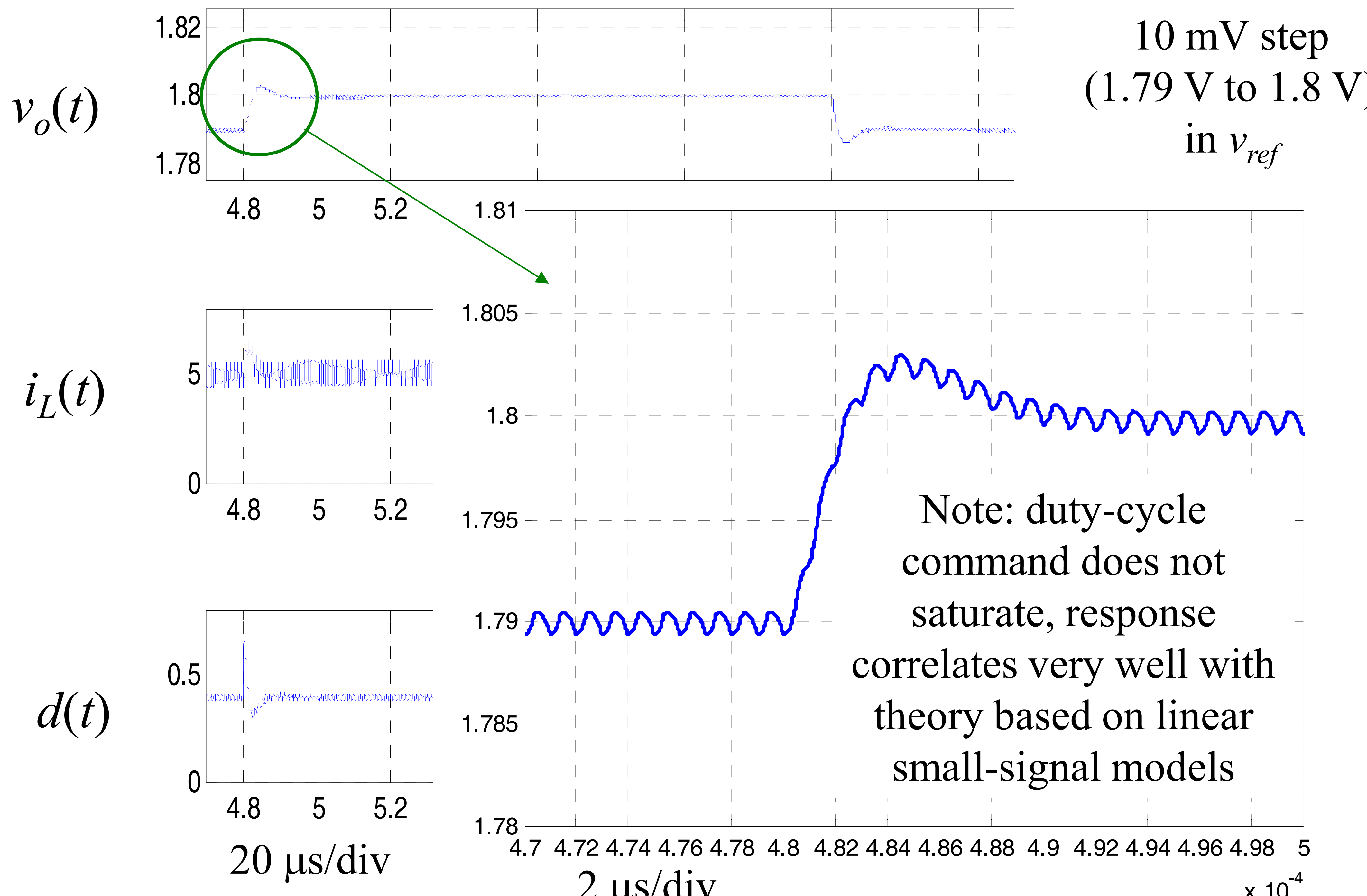


Small-signal step-reference response



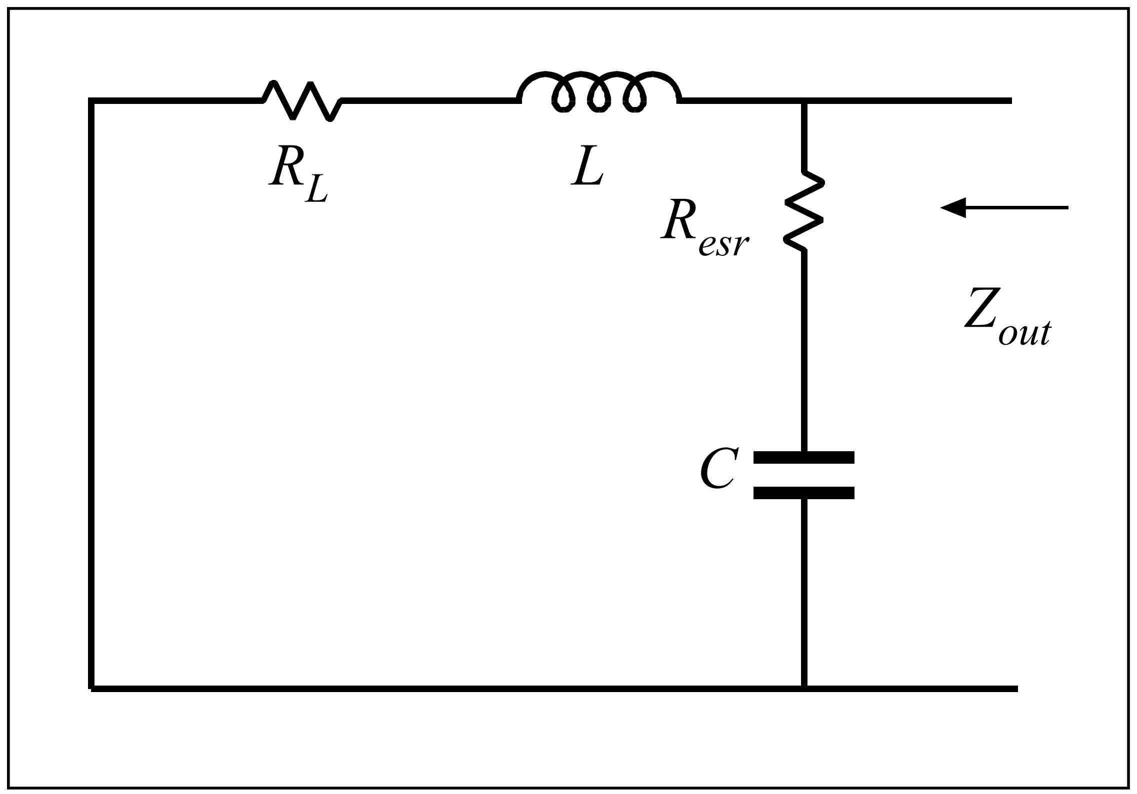
10 mV step
(1.79 V to 1.8 V)
in v_{ref}

Small-signal step-reference response



Output impedance

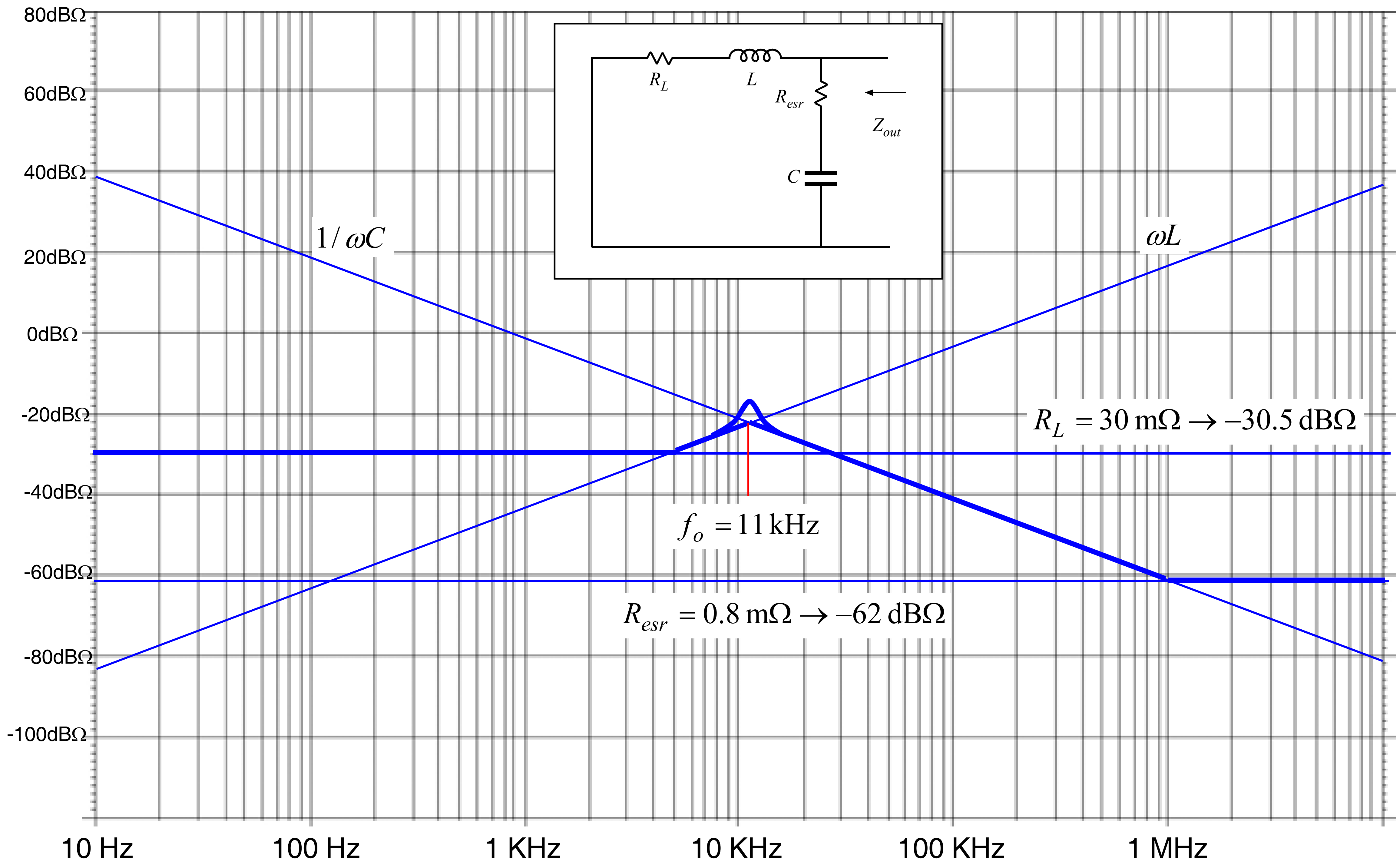
Synchronous buck open-loop output impedance



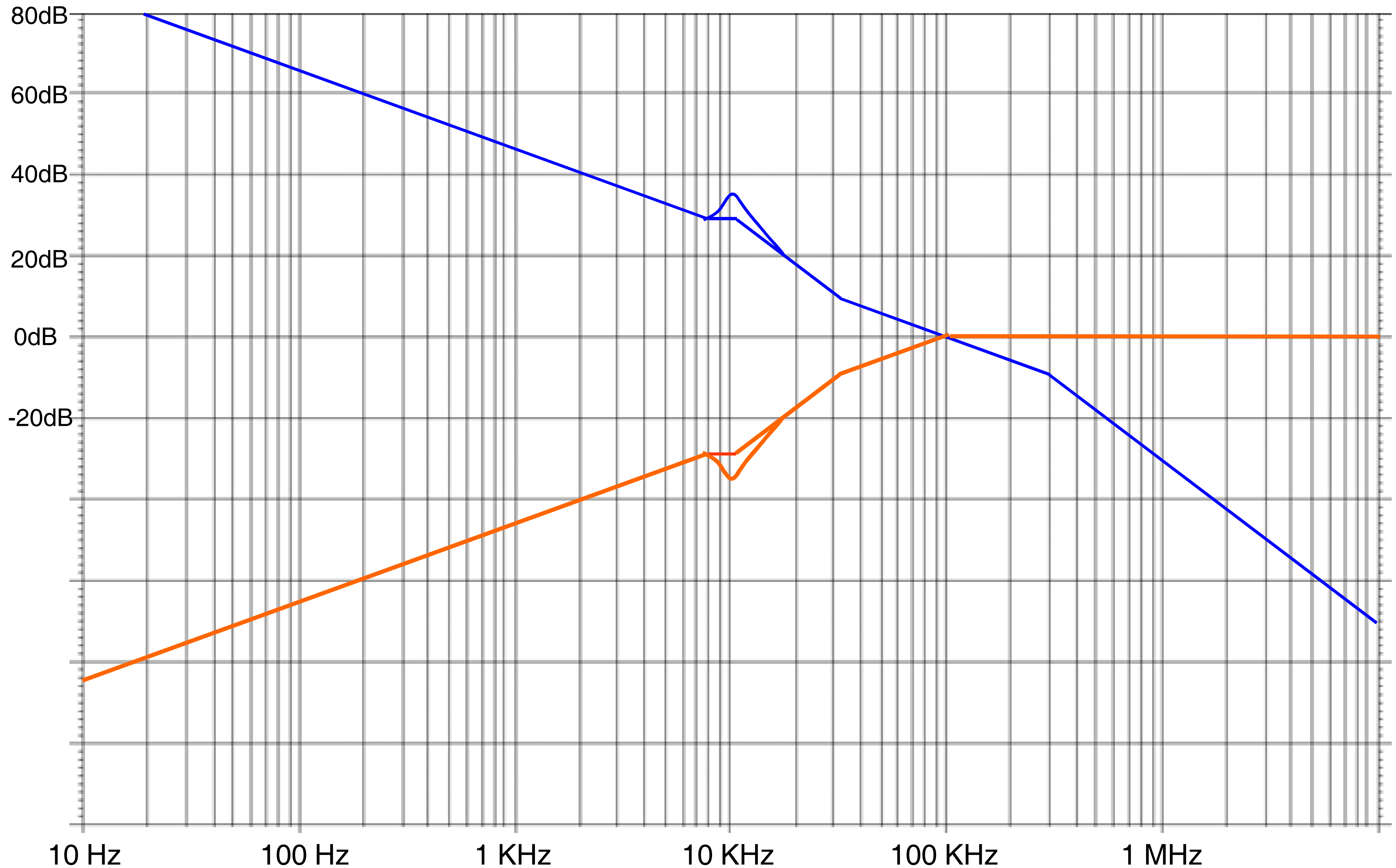
$$Z_{out}(s) = \left(R_{esr} + \frac{1}{sC} \right) \| \left(R_L + sL \right)$$

- L = 1 μH
- R_L = 30 mΩ
- C = 200 μF
- R_{esr} = 0.8 mΩ

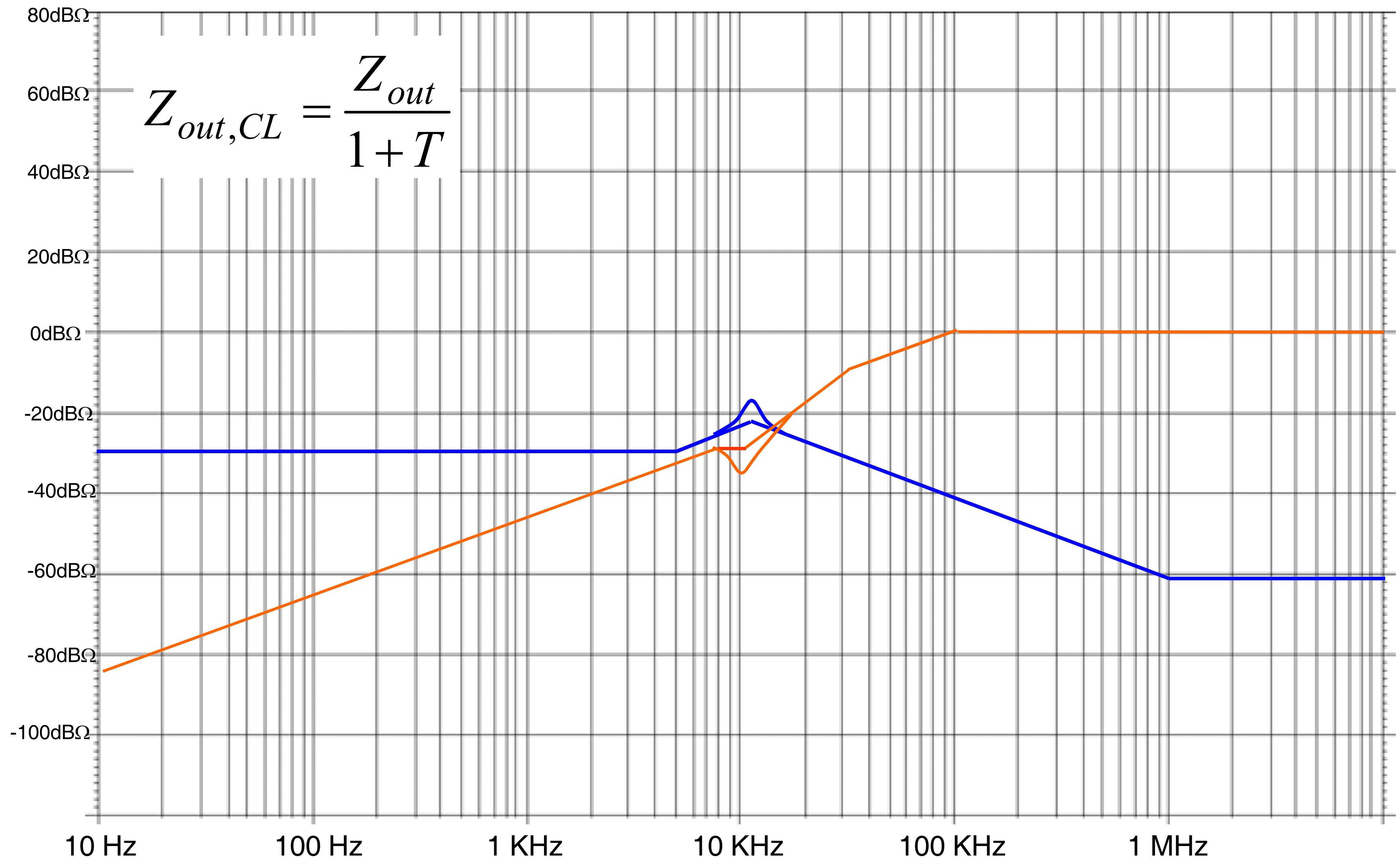
Open-loop output impedance: algebra on the graph



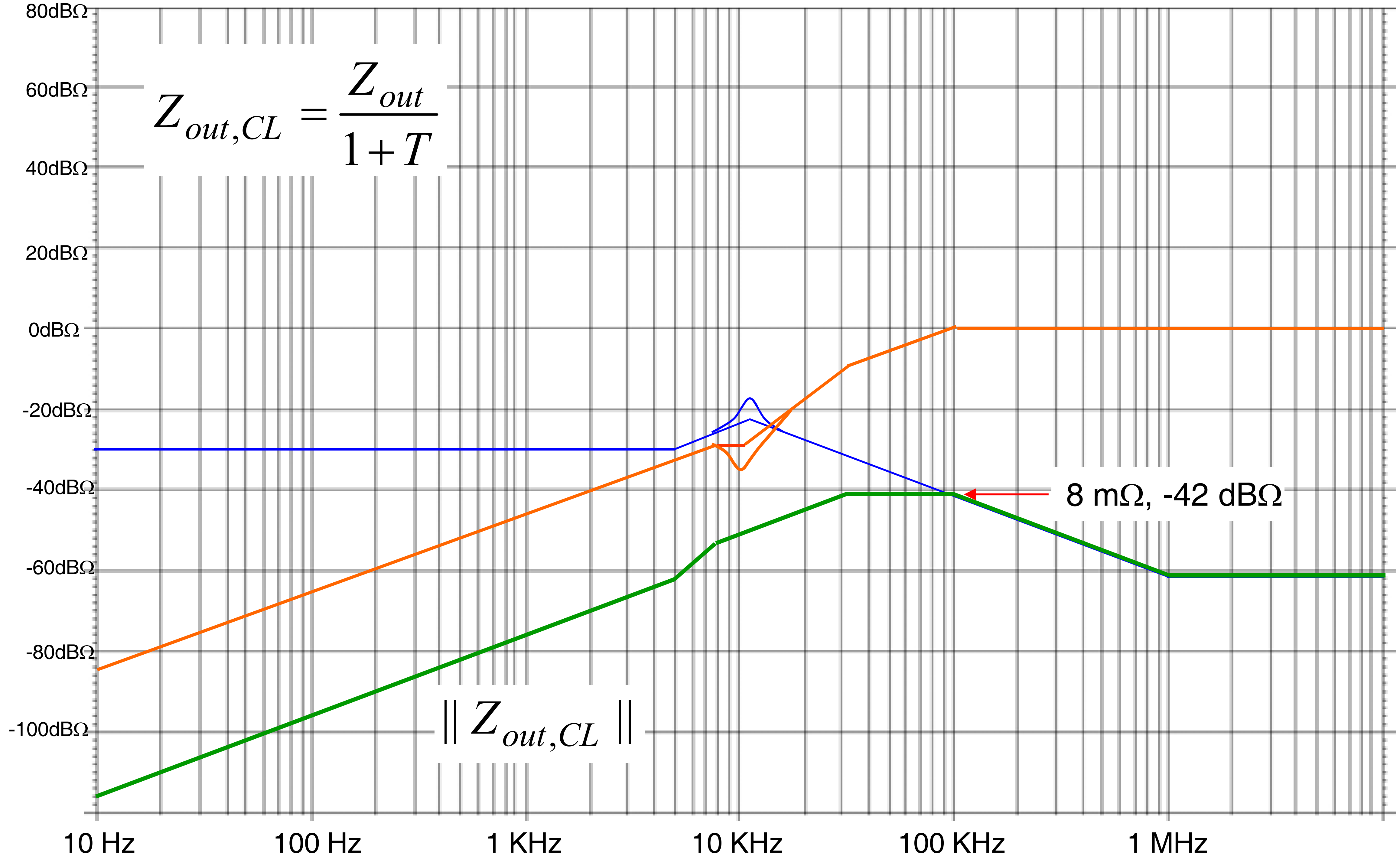
Construction of $1/(1+T)$



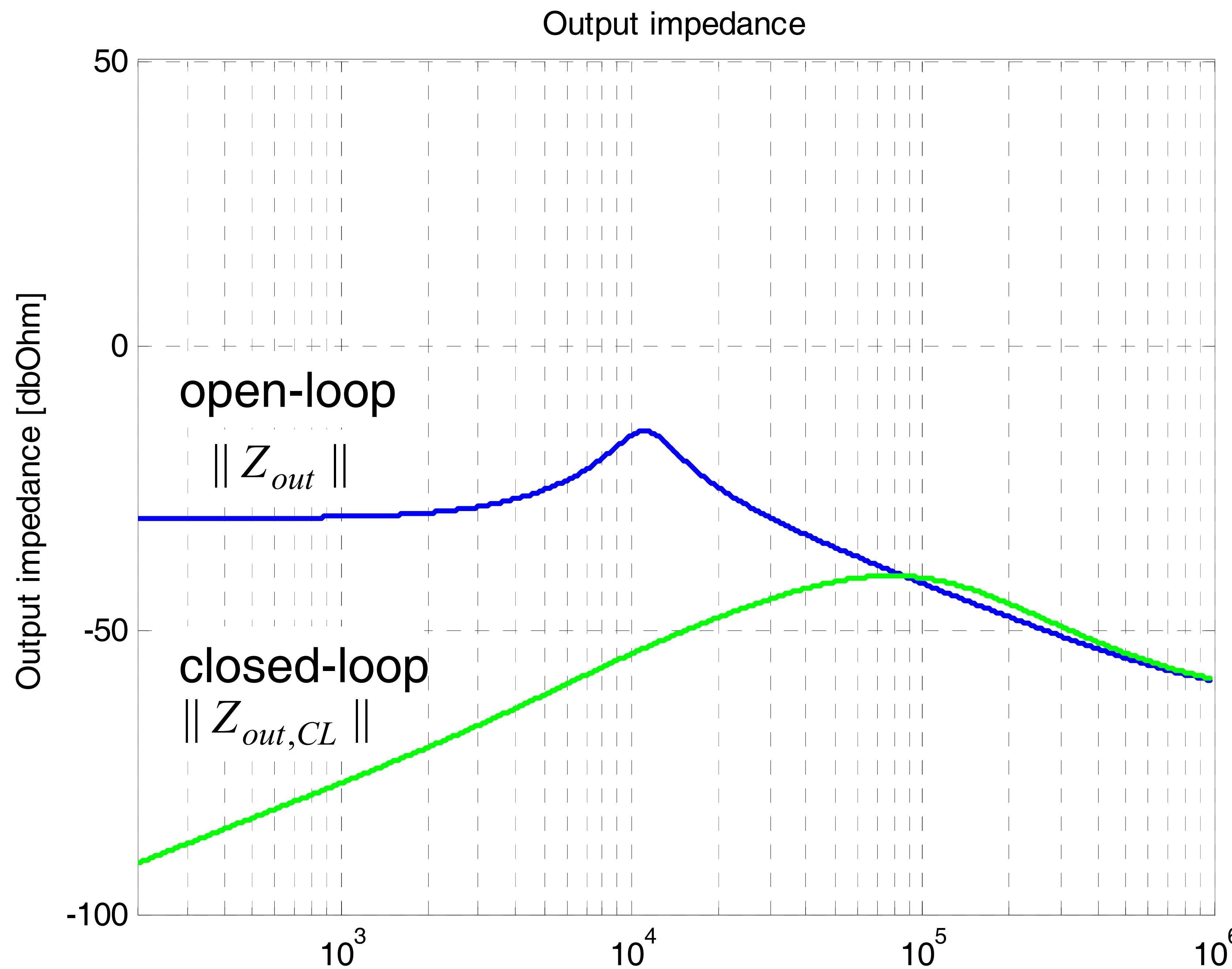
Construction of closed-loop output impedance



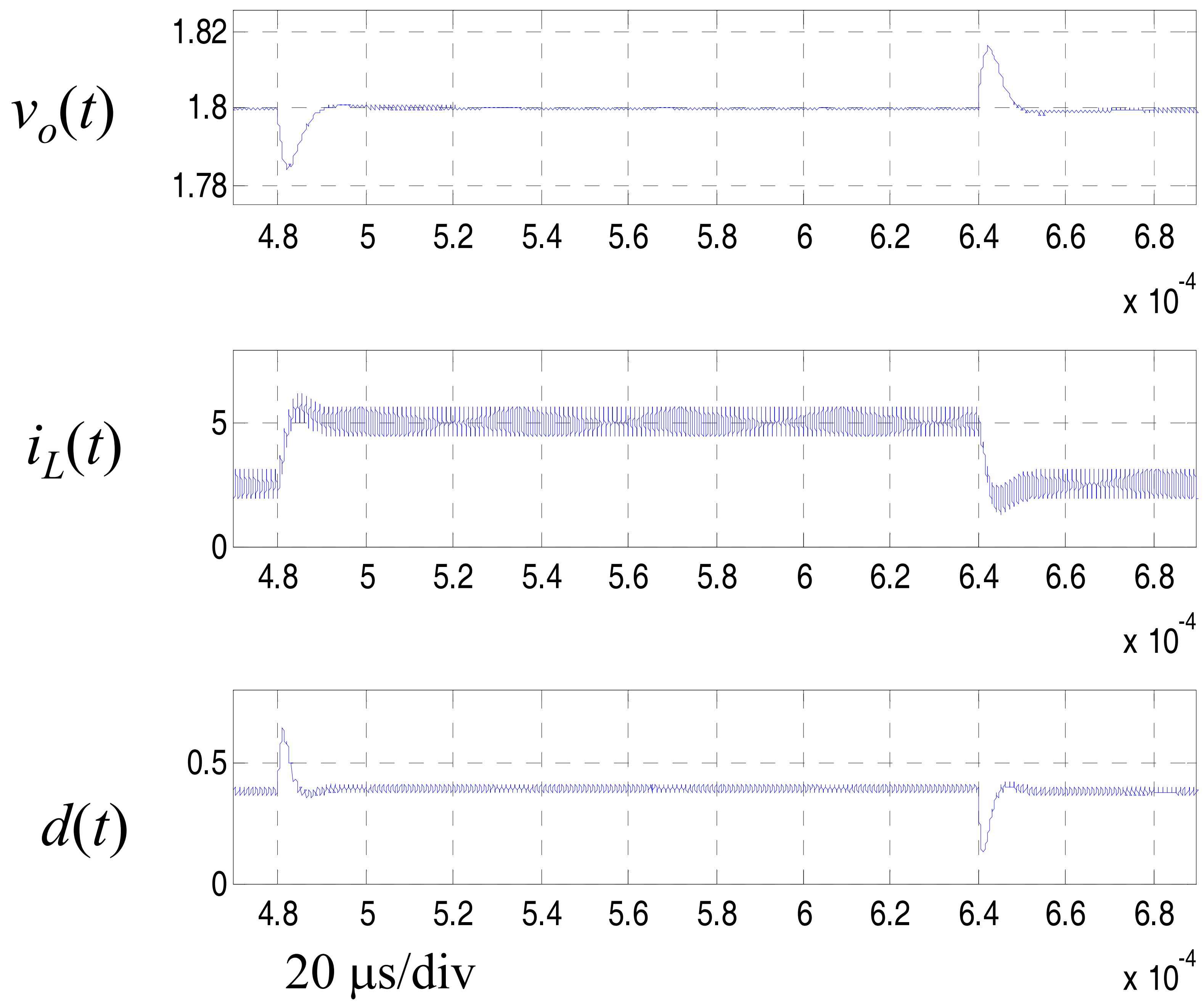
Closed-loop output impedance $Z_{out,CL}$



Verification: closed-loop output impedance



Step-load transient responses



2.5-5 A step-load transient

Step-load transient responses